Subject Code: YC MATHS 10	YASHWANT CLASSES Estd.1995	Subject: MATHS

Answer to this Paper must be written on the paper provided separately.
You will not be allowed to write during first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.
Section A is compulsory. Attempt any four questions from Section B.
The intended marks for questions or parts of questions are given in brackets [].

SECTION A (40 MARKS)
(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: [15]

(i) A shopkeeper bought an LED for ₹24,000 from a dealer. He sold it to a consumer at a profit of ₹6000. If the rate of GST is 28%, then the tax liability of the shopkeeper is :

(a) ₹1680 (b) ₹1800 (c) ₹2000 (d) ₹2420

(ii) The roots of the quadratic equation $3x^2 - 2x + 1 = 0$ are :

(a) real and distinct (b) rational and unequal (c) real and equal (d) not real

(iii) On dividing $f(x)$ by $(2x + 3)$, the remainder is :

(a) 0 (b) $f\left(\frac{2}{3}\right)$ (c) $f\left(\frac{-3}{2}\right)$ (d) $f\left(\frac{-2}{3}\right)$

(iv) Which of the following is a diagonal matrix?

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 5 & 4 \\ 1 & 0 & 0 \end{bmatrix}$ (d) none of these

(v) 10% ₹100 shares at ₹120 means annual dividend on 1 share is:

(a) ₹10 (b) ₹12 (c) ₹2 (d) none of these

(vi) The reflection of the point P(–2, 3) in the y-axis is:

(a) (2, 3) (b) (2, –3) (c) (–2, –3) (d) (–2, 3)

(vii) If $\Delta ABC \sim \Delta DEF$, then $\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF}$ is :

(a) AB × DE (b) $\frac{BC}{EF}$ (c) $\frac{AC}{DE}$ (d) $\frac{AB}{EF}$

(viii) The radii of two circular cylinders are in the ratio 4 : 5, and their heights are in the ratio 5 : 3. The ratio of their volumes is:

(a) 8 : 5 (b) 16 : 15 (c) 17 : 13 (d) 13 : 8

(ix) The 5th term from the end of the GP 2, 6, 18, ... 13122 is:

(a) 54 (b) 162 (c) 486 (d) 1458

(x) A bag contains 4 red and 8 blue marbles. A marble is drawn at random. The probability of drawing a red marble is:

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

(xi) In a size transformation with scale factor k, the area of the given figure is

(a) $k \times$ area of the resulting figure

(b) $k^2 \times$ area of the resulting figure

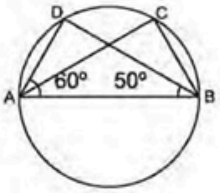
(c) $\frac{1}{k} \times$ area of the resulting figure

(d) $\frac{1}{k^2} \times$ area of the resulting figure

(xii) The point P(1, 2) divides the join of A(–2, 1) and B(7, 4) in the ratio:

(a) 1 : 2 (b) 2 : 1 (c) 3 : 2 (d) 2 : 3

(xiii) In the given figure, if $\angle DAB = 60°$ and $\angle ABD = 50°$, then $\angle ACB$ is equal to :



(a) 60° (b) 50°

(c) 70° (d) 80°

(xiv) The sum of first 10 terms of the AP whose nth term is $T_n = 2n + 1$, is :

(a) 100 (b) 120 (c) 125 (d) 130

(xv) The lower quartile of given observations 21, 14, 19, 18, 10, 15, 11 is:

(a) 10 (b) 11 (c) 14 (d) 15

Question 2 :

(i) Mrs. Bhatnagar has a 4 years time deposit account and deposits ₹650 per month. If she received ₹36,296 at the time of maturity, find the rate of interest. [4]

(ii) If a, b, c are in continued proportion, prove that $\frac{(a+b)^2}{(b+c)^2} = \frac{a}{c}$. [4]

(iii) By selling at ₹92 some $2\frac{1}{2}$% shares of face value ₹100 and investing the proceeds in 5% shares of face value ₹100 selling at ₹115, a person increased his income by ₹90 a year. Find . [4]

(a) the number of shares sold (b) the number of shares bought

(c) the original income

Question 3 :

(i) The curved surface area of a cylinder is 5,500 cm^2 and the circumference of the base is 110 cm. Find the height and volume of the cylinder. [4]

(ii) Find the equation of the straight line passing through the intersection of $2x + 5y = 4$ with x-axis and parallel to the line $3x - 7y = -8$. [4]

(iii) Use graph paper for this question. Take 1 cm = 1 unit on both axes.

(a) Plot the points P (2, 3) and Q (3, 1).

(b) Reflect P in x-axis to P′. Reflect P′ in y-axis to P″. Write coordinates of P′ and P″.

(c) Reflect Q in y-axis to Q′ and reflect Q′ in the origin to Q″. Write coordinates of Q′ and Q″.

(d) Write the geometrical name of PQQ″P′. [5]

SECTION B
(Attempt any four questions)

Question 4 :

(i) A dealer purchased goods for ₹16,50,000 and sold them for ₹20,00,000 within the state. If the rate of GST is 12%, find the net CGST and SGST payable by the dealer. [3]

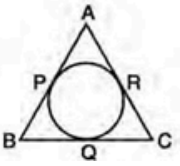
(ii) Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures. [3]

(iii) If pth, qth and rth terms of a GP be a, b, c respectively, then prove that $a^{q-r} \times b^{r-p} \times c^{p-q} = 1$. [4]

Question 5 :

(i) If $A = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$, find a matrix X such that $X + A = B$. [3]

(ii) In the figure, the incircle of ΔABC touches the sides AB, BC and CA at the points P, Q, R respectively.



Show that AP + BQ + CR = BP + CQ + AR

$= \frac{1}{2}$ (perimeter of ΔABC). [3]

(iii) If the expression $x^3 + ax + b$ is divided by $(x + 1)$ and $(x + 2)$, it gives the same remainder 12. Find the values of a and b. [4]

Question 6 :

(i) Find the gradient and the intercept on the y-axis of the line $3y + 2x - 12 = 0$. [3]

(ii) Show that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$ [3]

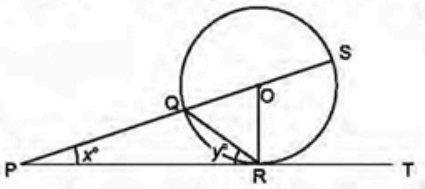
(iii) Find the sum $(-5) + (-8) + (-11) + \ldots (-62)$. [4]

Question 7 :

(i) The king, queen and jack of clubs are removed from a deck of playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of heart. [3]

(ii) The sides of a rectangular plot of land in a map were 12 cm and 5 cm. The map was drawn to scale 1 : 2000. Find the (a) diagonal of the plot in m (b) area of the plot in m^2. [3]

(iii) In the figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P. If $\angle SPR = x°$, and $\angle QRP = y°$, show that $x° + 2y° = 90°$. [4]



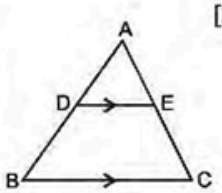
Question 8 :

(i) Solve the inequation : $8 < 5(x + 1) - 2 \leq 18$, $x \in R$. Graph the solution set. [3]

(ii) Find the mean of the following data : [3]

Class	0–10	10–20	20–30	30–40	40–50
Frequency	12	16	6	7	9

(iii) In the figure, DE ∥ BC. If AD = 3.4 cm, AB = 8.5 cm, and AC = 13.5 cm, find AE. [4]



Question 9 :

(i) Draw a circle of radius 4 cm and mark two chords AB and AC of the circle of length 6 cm and 5 cm respectively. [4]

(a) Construct the locus of points inside the circle that are equidistant from A and C.

(b) Construct the locus of points inside the circle that are equidistant from AB and AC.

(ii) Weight of 100 students is recorded below :

Weight in kg	30–35	35–40	40–45	45–50	50–55	55–60
No. of students	4	16	40	22	10	8

Draw an ogive and hence estimate the median. [6]

Question 10 :

(i) If $\frac{7m + 2n}{7m - 2n} = \frac{5}{3}$, use properties of proportion to find, $m : n$ [3]

(ii) Construct a triangle ABC in which AB = 6 cm, BC = 4 cm and AC = 5.5 cm.
Draw the incircle of the triangle. [3]

(iii) From the top of a building 20 m high, the angle of elevation of the top of a monument is 45° and the angle of depression of its foot is 15°. Find the height of the monument. [4]

Solution :

(i) (a) Disered tax liability = 28% of ₹6000 = ₹1680.

(ii) (d) Here $D = (-2)^2 - 4 \times 3 \times 1 = -8 < 0$

So, the roots are not real.

(iii) (c) $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$. So, required remainder is $f\left(\frac{-3}{2}\right)$

(iv) (b)

(v) (a) Annual dividend = 10% of ₹100 = ₹10

(vi) (a) $R_y(-2, -3) = (2, 3)$

(vii) (b)

(viii) (b) Required ratio $= \frac{\pi \times (4x)^2 \times 5y}{\pi \times (5x)^2 \times 3y} = \frac{16 \times 5}{25 \times 3} = 16 : 15.$

(ix) (b) Required term $= \frac{13122}{3^4} = 162.$

(x) (c) Required probability $= \frac{4}{12} = \frac{1}{3}$

(xi) (d)

(xii) (a) Let the required ratio be $m : 1$; Then, $1 = \frac{m \times 7 + 1 \times (-2)}{m + 1}$

$\Rightarrow m + 1 = 7m - 2 \Rightarrow 6m = 3 = m = \frac{1}{2}$

$\therefore$ Required ratio is $\frac{1}{2} : 1 = 1 : 2.$

(xiii) (c) $\angle ADB = 180^\circ - (60^\circ + 50^\circ) = 70^\circ$

$\angle ACB = \angle ADB = 70^\circ$

(xiv) (b) $d = T_2 - T_1 = 5 - 3 = 2 \Rightarrow S_{10} = \frac{10}{2}[2 \times 3 + (10 - 1) \times 2] = 120.$

(xv) (a) Here, $n = 8$, so median $= \frac{1}{2}\left[\text{value of } \left(\frac{8}{2}\right)^{\text{th}} \text{ observation} + \text{value of } \left(\frac{8}{2}+1\right)^{\text{th}} \text{ observation}\right]$

$= \frac{1}{2}(24 - x + 22 + 2x) = \frac{1}{2}(46 + x)$

$\Rightarrow 24 = \frac{1}{2}(46 + x) \Rightarrow 46 + x = 48 \Rightarrow x = 2.$

Question 2 :

(i) Mrs. Bhatnagar has a 4 years time deposit account and deposits ₹650 per month. If she received ₹36296 at the time of maturity, find the rate of interest. [4]

(ii) If a, b, c are in continued proportion, prove that $\frac{(a+b)^2}{(b+c)^2} = \frac{a}{c}$. [4]

(iii) By selling at ₹92 some $2\frac{1}{2}$% shares of face value ₹100 and investing the proceeds in 5% shares of face value ₹100 selling at ₹115, a person increased his income by ₹90 a year. Find . [4]

(a) the number of shares sold (b) the number of shares bought

(c) the original income

Solution : (i) $n = 48$ months, P = ₹ 650

$\therefore$ S. I.$= P \times \frac{n(n+1)}{2} \times \frac{r}{100} \times \frac{1}{12} =$ ₹ $\left\{650 \times \frac{48 \times 49}{2} \times \frac{r}{100} \times \frac{1}{12}\right\} =$ ₹ $637r$

$\therefore$ Total amount received on maturity = ₹ $(48 \times 650 + 637r)$

But, $48 \times 650 + 637r = 36296$

$\Rightarrow 637r = 36296 - 31200 = 5096 \Rightarrow r = \frac{5096}{637} = 8$

Hence, rate of interest = 8% **Ans.**

(ii) Let a, b and c are in continued proportion. Then

$\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac$

Now, LHS $= \frac{(a+b)^2}{(b+c)^2} = \frac{a^2+b^2+2ab}{b^2+c^2+2bc} = \frac{a^2+ac+2ab}{ac+c^2+2bc} = \frac{a(a+c+2b)}{c(a+c+2b)} = \frac{a}{c}$ = RHS **Proved.**

(iii) (a) Let the number of shares sold = x

$\therefore$ Sale proceeds = ₹92 × x = ₹92x

$\therefore$ Number of shares bought = $\frac{92x}{115}$

$\therefore$ New income = ₹$\frac{92x}{115} \times 5$ = ₹ $\frac{92x}{23}$

Original income = ₹$2\frac{1}{2} \times x$ = ₹ $\frac{5}{2}x$

But, $\frac{92x}{23} - \frac{5}{2}x = 90 \Rightarrow \frac{184x - 115x}{46} = 90 \Rightarrow \frac{69x}{46} = 90 \Rightarrow x = 90 \times \frac{46}{69} = 60$

$\therefore$ Number of shares sold = 60 **Ans.**

(b) Number of shares bought = $\frac{92x}{115} = \frac{92 \times 60}{115}$ = 48 **Ans.**

(c) Original income = ₹$\frac{5}{2}x$ = ₹$\frac{5}{2} \times 60$ = ₹150 **Ans.**

Question 3 :

(i) The curved surface area of a cylinder is 5,500 cm^2 and the circumference of the base is 110 cm. Find the height and volume of the cylinder. [4]

(ii) Find the equation of the straight line passing through the intersection of $2x + 5y = 4$ with x-axis and parallel to the line $3x - 7y = -8$. [4]

(iii) Use graph paper for this question. Take 1 cm = 1 unit on both axes.

(a) Plot the points P (2, 3) and Q (3, 1).

(b) Reflect P in x-axis to P′. Reflect P′ in y-axis to P″. Write coordinates of P′ and P″.

(c) Reflect Q in y-axis to Q′ and reflect Q′ in the origin to Q″. Write coordinates of Q′ and Q″.

(d) Write the geometrical name of PQQ″P′. [5]

Solution :

(i) $2\pi r = 110 \Rightarrow r = \frac{110 \times 7}{2 \times 22}$ = 17.5 cm

Also, curved surface area of the cylinder = $2\pi rh$

$\Rightarrow 5{,}500 = 2\pi rh \Rightarrow 5{,}500 = 110 \times h \Rightarrow h = \frac{5{,}500}{110}$ = 50 cm **Ans.**

$\therefore$ Volume of the cylinder = $\pi r^2 h = \frac{22}{7} \times 17.5 \times 17.5 \times 50$ cm^3 = 48,125 cm^3 **Ans.**

(ii) Given line is $2x + 5y = 4$; Equation of x-axis is $y = 0$.

$\therefore\ 2x + 0 = 4 \Rightarrow x = 2$

$\Rightarrow$ Coordinates of the point of intersection of $2x + 5y = 4$ and x-axis are (2, 0).

Let the equation of the required line be $y = mx + c$ ....(1)

$\because$ (1) passes through (2, 0)

$\therefore\ 0 = 2m + c \Rightarrow 2m + c = 0$ .... (2)

Again, (1) is parallel to $3x - 7y = -8$

or $y = \frac{3x}{7} + \frac{8}{7}$

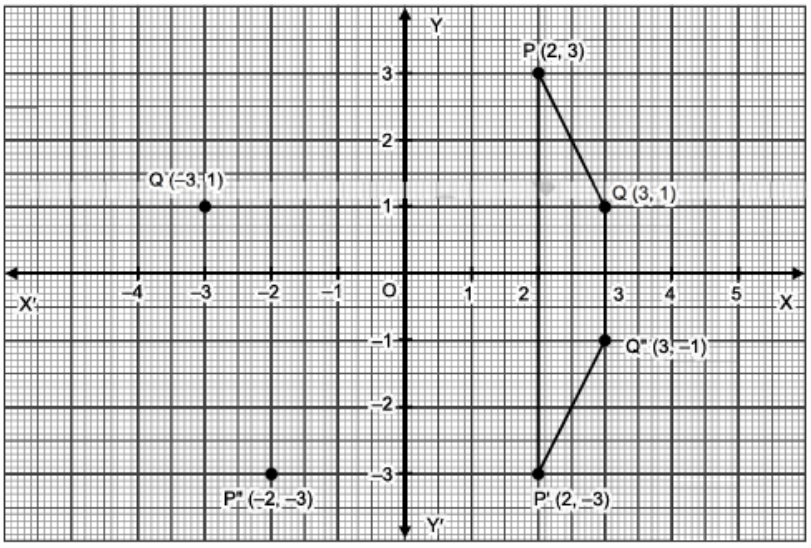
$\therefore$ Gradient m of (1) $= \frac{3}{7}$

Putting $m = \frac{3}{7}$ in (2), we have, $2 \times \frac{3}{7} + c = 0 \Rightarrow c = -\frac{6}{7}$

Substituting $m = \frac{3}{7}$ and $c = -\frac{6}{7}$ in (1), we get, $y = \frac{3}{7}x - \frac{6}{7}$

$\Rightarrow 7y = 3x - 6 \Rightarrow 3x - 7y = 6$, which is the required equation. **Ans.**

(iii)



(a) The given points have been plotted.

(b) Coordinates of P′ are (2, –3) and coordinates of P″ are (–2, –3) **Ans.**

(c) Coordinates of Q′ and Q″ are respectively (–3, 1), and (3, –1). **Ans.**

(d) PQQ″P′ is an isosceles trapezium. **Ans.**

Question 4 :

(i) A dealer purchased goods for ₹16,50,000 and sold them for ₹20,00,000 within the state. If the rate of GST is 12%, find the net CGST and SGST payable by the dealer. [3]

(ii) Solve the quadratic equation $x^2 - 3(x + 3) = 0$; Give your answer correct to two significant figures. [3]

(iii) If pth, qth and rth terms of a GP be a, b, c respectively, then prove that $a^{q-r} \times b^{r-p} \times c^{p-q} = 1$. [4]

Solution :

(i) Purchase price for the dealer = ₹16,50,000

$\therefore$ Input GST = CGST + SGST = 6% of ₹16,50,000 + 6% of ₹16,50,000 = ₹99,000 + ₹99,000

Selling price = ₹20,00,000

$\therefore$ Output GST = CGST + SGST

= 6% of ₹20,00,000 + 6% of ₹20,00,000

= ₹1,20,000 + ₹1,20,000

Net CGST paid by the dealer = ₹(1,20,000 – 99,000) = ₹21,000 **Ans.**

Net SGST paid by the dealer = ₹(1,20,000 – 99,000) = ₹21,000 **Ans.**

(ii) We have, $x^2 - 3(x + 3) = 0 \Rightarrow x^2 - 3x - 9 = 0$

$$\Rightarrow x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-9)}}{2 \times 1} \Rightarrow x = \frac{3 \pm \sqrt{9 + 36}}{2} \Rightarrow x = \frac{3 \pm 6.70}{2}$$

$$\Rightarrow x = \frac{9.70}{2} \text{ or } x = \frac{-3.70}{2} \quad \Rightarrow x = 4.85 \text{ or } x = -1.85$$

$\Rightarrow x = 4.9$ or $x = -1.9$ **Ans.**

(iii) Let A be the first term and R be the common ratio of the given GP.

Then, $T_p = a \Rightarrow AR^{p-1} = a$

$T_q = b = AR^{q-1} = b$

$T_r = c = AR^{r-1} = c$

Now, $a^{q-r} \times b^{r-p} \times c^{p-q} =$

$= (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q}$

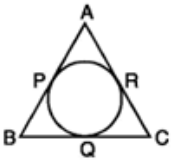
$= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$

$= A^0 \times R^0 = 1$ Proved.

Question 5 :

(i) If $A = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$, find a matrix X such that X + A = B. **[3]**

(ii) In the figure, the incircle of ΔABC touches the sides AB, BC and CA at the points P, Q, R respectively.



Show that AP + BQ + CR = BP + CQ + AR

$$= \frac{1}{2} \text{ (perimeter of } \Delta ABC).$$ **[3]**

(iii) If the expression $x^3 + ax + b$ is divided by $(x + 1)$ and $(x + 2)$, it gives the same remainder 12. Find the values of a and b. **[4]**

Solution :

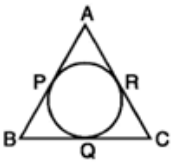
(i) $X + A = B \Rightarrow X + A + (-A) = B + (-A)$

$\Rightarrow X + [A + (-A)] = B + (-A)$

$\Rightarrow X + O = B + (-A)$

$$\Rightarrow X = B + (-A) \Rightarrow X = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ -3 & -6 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -5 & -1 \end{bmatrix}$$ **Ans.**

(ii) We know that the tangents drawn from an external point to a circle are equal.



$\therefore$ AP = AR, BQ = BP and CR = CQ

$\therefore$ AP + BQ + CR= AR + BP + CQ ...(1)

Now, perimeter of ΔABC = AB + BC + CA

= AP + PB + BQ + QC + CR + AR

= (AP + BQ + CR) + (PB + QC + AR)

= 2 (AP + BQ + CR) [From (1)]

$$\Rightarrow AP + BQ + CR = AR + BP + CQ = \frac{1}{2} \text{ (perimeter of } \Delta ABC)$$

$$\Rightarrow AP + BQ + CR = BP + CQ + AR = \frac{1}{2} \text{ (perimeter of } \Delta ABC)$$ **Proved.**

(iii) Let $f(x) = x^3 + ax + b$

On dividing $f(x)$ by $(x + 1)$, we have

$f(-1) = (-1)^3 + a(-1) + b = 12$

$\Rightarrow \quad -1 - a + b = 12$

$\Rightarrow \quad -a + b = 13 \quad \ldots. (1)$

Again dividing $f(x)$ by $(x + 2)$, we have

$f(-2) = (-2)^3 + a(-2) + b = 12$

$\Rightarrow \quad -8 - 2a + b = 12$

$\Rightarrow \quad -2a + b = 20 \quad \ldots (2)$

Subtracting (1) from (2), we get

$-a = 7 \quad \Rightarrow a = -7$

Substituting $a = -7$ in (1), we get

$7 + b = 13 \Rightarrow b = 6$

Hence, $a = -7, \ b = 6$ **Ans.**

Question 6 :

(i) Find the gradient and the intercept on the y-axis of the line $3y + 2x - 12 = 0$. **[3]**

(ii) Show that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$ **[3]**

(iii) Find the sum $(-5) + (-8) + (-11) + \ldots (-62)$. **[4]**

Solution :

(i) $3y + 2x - 12 = 0 \Rightarrow 3y = -2x + 12 \Rightarrow y = \left(-\frac{2}{3}\right)x + 4$

Comparing it with $y = mx + c$, we get $m = -\frac{2}{3}$ and $c = 4$

Gradient (slope) $m = -\frac{2}{3}$ and intercept on y-axis = 4 units. **Ans.**

(ii) LHS = $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \left\{\frac{(\sin A + \cos A) - 1}{\sin A}\right\}\left\{\frac{(\sin A + \cos A) + 1}{\cos A}\right\}$$

$$= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A} \quad [\because (a - b)(a + b) = a^2 - b^2]$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A} = \frac{2 \sin A \cos A}{\sin A \cos A} = 2 = \text{RHS} \quad \textbf{Proved.}$$

(iii) Here, $a = -5, \ d = -8 - (-5) = -3, \quad l = -62$

Let the given AP has n terms.

Then, $T_n = l = a + (n - 1)d$

$\Rightarrow -62 = -5 + (n - 1)(-3)$

$$\Rightarrow n - 1 = \frac{-62 + 5}{(-3)} = 19$$

$\Rightarrow n = 20$

$\therefore$ Sum of the given AP $= \frac{n}{2}(a + l) = \frac{20}{2}[-5 + (-62)] = 10 \times (-67) = -670.$ **Ans.**

Question 7 :

(i) Draw a circle of radius 4 cm and mark two chords AB and AC of the circle of length 6 cm and 5 cm respectively. [5]

(a) Construct the locus of points inside the circle that are equidistant from A and C.

(b) Construct the locus of points inside the circle that are equidistant from AB and AC.

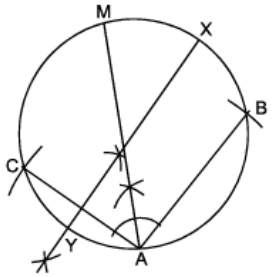
(ii) Weight of 100 students is recorded below :

Weight in kg	30–35	35–40	40–45	45–50	50–55	55–60
No. of students	4	16	40	22	10	8

Draw an ogive and hence estimate the median. [5]

Solution :

(i) (a) XY is the locus of points equidistant from A and C.



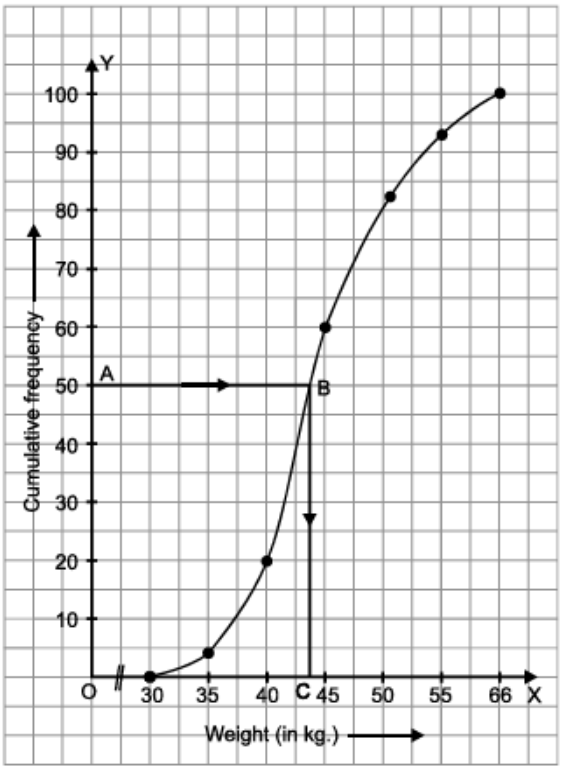
(b) MA is the locus of points equidistant from AB and AC.

(ii)

Weight (in kg)	No. of students	Cumulative frequency
30–35	4	4
35–40	16	20
40–45	40	60
45–50	22	82
50–55	10	92
55–60	8	100
	N = 100	

$N = 100 \Rightarrow \frac{N}{2} = 50$

Let A denote 50 on the y-axis. From A, draw AB parallel to x-axis, which meets the ogive at B. From B, draw BC $\perp$ x axis. The abscissa of C is 44. So, median = 44 **Ans.**

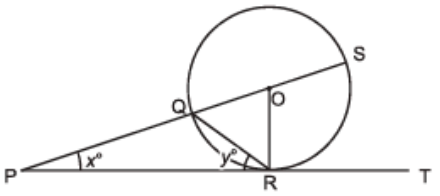


Question 8 :

(i) The king, queen and jack of clubs are removed from a deck of playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of heart. [3]

(ii) The sides of a rectangular plot of land in a map were 12 cm and 5 cm. The map was drawn to scale 1 : 2000. Find the (a) diagonal of the plot in m (b) area of the plot in m^2. [3]

(iii) In the figure, PT touches a circle with centre O at R. Diameter SQ when produced meets PT at P. If $\angle SPR = x^o$ and $\angle QRP = y^o$, show that $x^o + 2y^o = 90^o$. [4]



Solution :

(i) Total number of outcomes = 52 – 3 = 49

Number of favourable outcomes = 13

$\therefore$ P(a heart) = $\frac{\text{Number of favourable outcomes}}{\text{total number of outcomes}} = \frac{13}{49}$ **Ans.**

(ii) Here the scale factor, $k = \frac{1}{2000}$

(a) Diagonal length on the map = $\sqrt{12^2 + 5^2} = \sqrt{169}$ = 13 cm.

Diagonal length of the plot = $\frac{1}{k}$ × diagonal length on the map

= 2000 × 13 cm = 260 m. **Ans.**

(b) Area of the plot = $\frac{1}{k^2}$ × area on the map = $(2000)^2 \times 12 \times 5$ cm^2

$= \frac{(2000)^2 \times 12 \times 5}{10000}$ m^2 = 24,000 m^2 **Ans.**

(iii) $\angle PRO = 90°$ [Radius through the point of contact is perpendicular to the tangent]

$\Rightarrow \angle ORQ = 90° - y°$

Also, OR = OQ [Radii of the same circle]

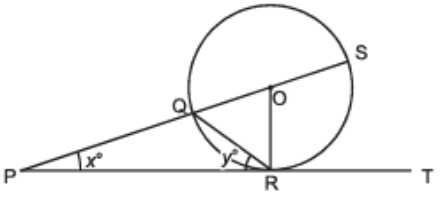
$\Rightarrow \angle OQR = \angle ORQ = 90° - y°$

In ΔPQR, $\angle OQR = \angle QPR + \angle QRP$

[Exterior angle = sum of two interior opposite angles]

$\Rightarrow 90° - y° = x° + y°$

$\Rightarrow x° + 2y° = 90°$ **Proved.**



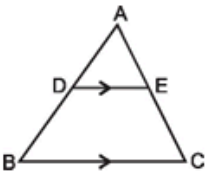
Question 9 :

(i) Solve the inequation : $8 < 5(x + 1) - 2 \leq 18$, $x \in R$. Graph the solution set. [3]

(ii) Find the mean of the following data : [3]

Class	0–10	10–20	20–30	30–40	40–50
Frequency	12	16	6	7	9

(iii) In the figure, DE || BC. If AD = 3.4 cm, AB = 8.5 cm, and AC = 13.5 cm, find AE. [4]



Solution :

(i) $8 < 5(x + 1) - 2 \leq 18$, $x \in R$

$\Rightarrow 8 < 5(x + 1) - 2$ and $5(x + 1) - 2 \leq 18$, $x \in R$

$\Rightarrow 8 < 5x + 5 - 2$ and $5x + 5 - 2 \leq 18$, $x \in R$

$\Rightarrow 8 - 3 < 5x$ and $5x \leq 18 - 3$, $x \in R$

$\Rightarrow$ $1 < x$ and $x \le 3$, $x \in R \Rightarrow 1 < x \le 3$, $x \in R$

$\therefore$ Solution set = $\{x : 1 < x \le 3,\ x \in R\}$

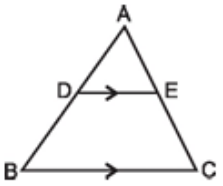
Graph of the solution set is shown below :

(ii)

Class	Frequency (f_i)	Class marks (x_i)	$f_i x_i$
0–10	12	5	60
10–20	16	15	240
20–30	6	25	150
30–40	7	35	245
40–50	9	45	405
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 1100$

Mean $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{1100}{50} = 22$ **Ans.**

(iii) Since, DE || BC, then by Basic Proportionality Theorem, we have



$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{AB - AD} = \frac{AE}{AC - AE}$$

$$\Rightarrow \frac{3.4}{8.5 - 3.4} = \frac{AE}{13.5 - AE} \Rightarrow \frac{3.4}{5.1} = \frac{AE}{13.5 - AE}$$

$$\Rightarrow \frac{2}{3} = \frac{AE}{13.5 - AE} \Rightarrow 3AE = 27 - 2AE \Rightarrow AE = \frac{27}{5} = 5.4 \text{ cm}$$ **Ans.**

Question 10 :

(i) If $\dfrac{7m + 2n}{7m - 2n} = \dfrac{5}{3}$, use properties of proportion to find, $m : n$ **[3]**

(ii) Construct a triangle ABC in which AB = 6 cm, BC = 4 cm and AC = 5.5 cm. Draw the incircle of the triangle. **[3]**

(iii) From the top of a building 20 m high, the angle of elevation of the top of a monument is 45° and the angle of depression of its foot is 15°. Find the height of the monument. **[4]**

Solution :

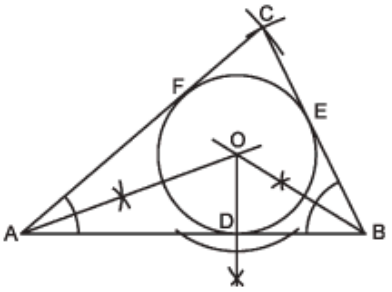
(i) We have, $\dfrac{7m + 2n}{7m - 2n} = \dfrac{5}{3}$

$$\Rightarrow \frac{7m + 2n + 7m - 2n}{7m + 2n - 7m + 2n} = \frac{5 + 3}{5 - 3}$$ [Applying componendo and dividendo]

$$\Rightarrow \frac{14m}{4n} = \frac{8}{2} \Rightarrow \frac{7m}{2n} = \frac{4}{1} \Rightarrow \frac{m}{n} = \frac{4}{1} \times \frac{2}{7} = \frac{8}{7} \Rightarrow m : n = 8 : 7$$ **Ans.**

(ii) **Steps of Construction :**

1. Draw AB = 6 cm.
2. With A and B as centres and radii equal to 5.5 cm and 4 cm respectively, draw two arcs. These arcs cut each other at C.
3. Join AC and BC to get the required triangle.
4. Draw the bisectors of $\angle A$ and $\angle B$. The bisectors of these angles meet at O.
5. From O, draw perpendicular on AB, which meets AB at D.
6. With O as centre and OD as radius, draw a circle, which touches the sides AB, BC and AC at D, E and F respectively.



(iii) Let AB be the building and CD be the monument.

Let CB = x m and DE = h m.

$\therefore$ AE = CB = x m

In ΔADE, $\frac{h}{x} = \tan 45° \Rightarrow h = x$ ...(1)

In ΔABC, $\frac{20}{x} = \tan 15°$

$\Rightarrow x = \frac{20}{\tan 15°} = \frac{20}{0.2679°} = 74.64$

$\Rightarrow h = 74.64$ [From (1)]

Hence, height of the monument = (20 + 74.64) m = 94.64 m **Ans.**