Subject Code: YC MATHS 9	YASHWANT CLASSES Estd.1995	Subject: MATHS

Answer to this Paper must be written on the paper provided separately.
You will not be allowed to write during first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.
Section A is compulsory. Attempt any four questions from Section B.
The intended marks for questions or parts of questions are given in brackets [].

SECTION A (40 MARKS)
(Attempt all questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: [15]

(i) The nominal value of a share :
(a) remains the same
(b) changes from time to time
(c) can only be changed by the share holder
(d) none of these

(ii) Roots of the equation $(x-1)^2 - 5(x-1) - 6 = 0$ are :
(a) 7, 0 (b) 6, 0 (c) 7, 6 (d) 6, −7

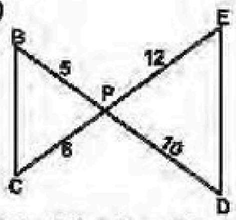
(iii) If $(x-2)$ is a factor of $x^2 + 5x + p$, then the value of p is :
(a) 10 (b) 12 (c) −13 (d) −14

(iv) The common ratio of the GP $\frac{-3}{4}, \frac{1}{2}, \frac{-1}{3}, \frac{2}{9}...$ is:
(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

(v) 8th term of the AP 5, 8, 11, ... 38, from the end is :
(a) 22 (b) 20 (c) 17 (d) 16

(vi) The reflection of the point P(0, −1) in the x-axis is:
(a) (1, 0) (b) (−1, 0) (c) (0, 0) (d) (0, 1)

(vii) In the figure, BD and CE intersect each other at P. ΔPBC ~ ΔPDE by :
(a) AA similarity
(b) SAS similarity
(c) SSS similarity
(d) RHS similarity



(viii) If Himanshu reshaped a cone of heigh h cm and radius of base r cm into a cylinder, then which of the following options is always correct?
(a) Volume of cone = Volume of cylinder
(b) Surface area of cone = Surface area of cylinder
(c) Radius of cone = Radius of cylinder
(d) None of the above

(ix) If $-1 \leq 3 + 4x < 23, x \in R$, then :
(a) $\{-1 \leq x < 5, x \in R\}$ (b) $\{-1 < x \leq 5, x \in R\}$ (c) $\{-2 \leq x < 5, x \in R\}$ (d) $\{-2 < x \leq 5, x \in R\}$

(x) The probability of drawing a black face card from a deck of 52 playing cards is:
(a) $\frac{1}{12}$ (b) $\frac{3}{26}$ (c) $\frac{1}{2}$ (d) $\frac{1}{13}$

(xi) If M = [1, –2], N = $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, then : The matrix MN is :

(a) [4, 3] (b) [–4, 3] (c) [4 –3] (d) $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

(xii) The mid-point of the line segment joining the points A(–1, 4) and B(–3, –2) is:

(a) (–2, –3) (b) (1, 3) (c) –2, 1 (d) 2, 1

(xiii) In the given figure, O is the centre of the circle. If ∠ABC = 20°, then ∠AOC is equal to :

(a) 20° (b) 40° (c) 60° (d) 10°

(xiv) If k be the scale factor of a size transformation, then $k > 1$ means:

(a) enlargement (b) reduction (c) identity transformation (d) none of these

(xv) The mode of the given observations 5, 3, 2, 7, 5, 9, 3, 8, 5, is:

(a) 3 (b) 5 (c) 9 (d) 2

Question 2 :

(i) Amit deposits ₹1600 per month in a bank for 18 months in a recurring deposit account. If he gets ₹31,080 at the time of maturity, what is the rate of interest per annum? [4]

(ii) If $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b}$, prove that $ax + by + cz = 0$. [4]

(iii) A man invests ₹20,020 in buying shares of nominal value ₹26 at 10% premium. The dividend on the shares is 15% per annum. Calculate: [4]

(a) the number of shares he buys
(b) the dividend he receives annually
(c) the rate of interest he gets on his money.

Question 3 :

(i) A solid cylinder of radius 7 cm and height 14 cm is melted and recast into solid spheres each of radius 3.5 cm. Find the number of spheres formed. [4]

(ii) Find the equation of the line passing through (2, 4) and parallel to the line $3x + 5y - 15 = 0$. [4]

(iii) With the help of a graph paper, taking 1 cm = 1 unit along both x and y axis: [5]

(a) Plot points A (0, 3), B (2, 3), C (3, 0), D (2, –3), E (0, –3).
(b) Reflect points B, C and D on the y axis and name them as B', C' and D' respectively.
(c) Write the co-ordinates of B', C' and D'.
(d) Write the equation of line B'D'.
(e) Name the figure BCDD'C'B'B.

SECTION B
(Attempt any four questions)

Question 4 :

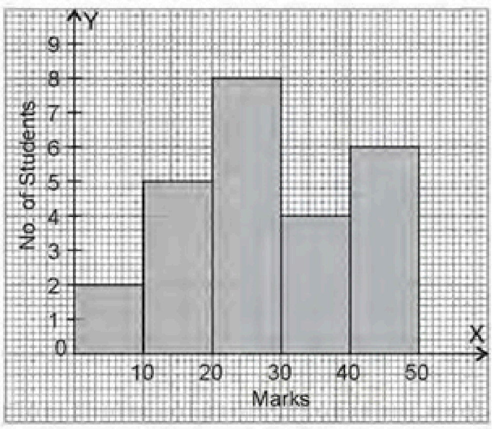
(i) A shopkeeper bought an article with market price ₹1200 from the wholesaler at a discount of 10%. The shopkeeper sells this article to the customer on the market price printed on it. If the rate of GST is 6%, then find : [3]

(a) GST paid by the wholesaler.
(b) Amount paid by the customer to buy the item.

(ii) Find the sum of first 8 terms of the GP $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots$ [3]

(iii) The histogram alongside represents the scores obtained by 25 students in a Mathematics mental test. Use the data to:

(a) Frame a frequency distribution table.
(b) To calculate mean.
(c) To determine the Modal class. [4]



Question 5 :

(i) Given $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$ [3]

Write down

(a) the order of the matrix X (b) the matrix X.

(ii) In the map of a rectangular plot of land, the length = 2.5 cm and breadth =1.4 cm. If the scale is 1:10000, find the area of the plot in m^2. [3]

(iii) Find the value of k if $4x^3 - 2x^2 + kx + 5$ leaves remainder -10 when divided by $2x + 1$. [4]

Question 6 :

(i) Determine whether the line through (–2, 3) and (4, 1) is perpendicular to the line $3x = y + 1$. Does the line $3x = y + 1$ bisect the line joining (–2, 3) and (4, 1)? [3]

(ii) Show that $\frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\sec A + 1}{\sec A - 1}$. [3]

(iii) The first term of an AP is 2 and the last term is 50. If the sum of all these terms is 442, find its common difference.[4]

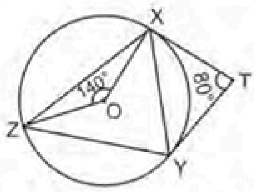
Question 7 :

(i) A box consists of 4 red, 5 black and 6 white balls. One ball is drawn out at random. Find the probability that the ball drawn is: [3]

(a) black (b) red or white

(ii) Draw a line segment AB = 3 cm. Draw the locus of a point P which moves at a distance of 5 cm from AB. [3]

(iii) In the figure, O is the centre of the circumcircle of $\angle XYZ$. Tangents at X and Y intersect at T. Given $\angle XTY = 80°$ and $\angle XOZ = 140°$, calculate the value of $\angle ZXY$. [4]



Question 8 :

(i) Solve the inequation $2x - \frac{5}{2} < x + \frac{3}{2} \leq 3x + \frac{11}{2}$, $x \in I$. Represent the solution set on a number line. [3]

(ii) The mean of the following distribution is 49. Find the missing frequency a. [3]

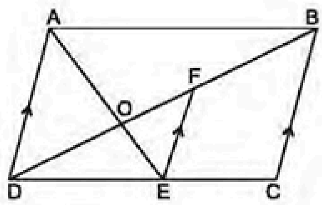
Class	0–20	20–40	40–60	60–80	80–100
Frequency	15	20	30	a	10

(iii) In the figure, ABCD is a parallelogram. E is the point on CD. AE intersects BD at O. EF ∥ CB. Find the following ratios, if DF : FB = 3 : 1. [4]

(a) ar(ΔDFE) : ar (ΔDBC)

(b) ar(ΔOFE) : ar (ΔODA)

(c) ar(ΔDFE) : ar (trap.EFBC)



Question 9 :

(i) An aeroplane travelled a distance of 400 km at an average speed of x km/h. On return journey, the speed was increased by 40 km/h. Write down an expression for the time taken for

(a) the onward journey (b) the return journey.

If the return journey took 30 minutes less than the onward journey, write down an equation in x and find its value [4]

(ii) The following table shows the distribution of marks in Mathematics:

Marks (less than)	No. of students
10	7
20	28
30	54
40	71
50	84
60	105
70	147
80	180

With the help of a graph paper, taking 2 cm = 10 units along one axis and 2 cm = 10 units along the other axis, draw an ogive for the above distribution and use it to find the:

(a) median.

(b) number of students who scored distinction marks (75% and above)

(c) number of students, who passed the examination if pass marks is 35%. [6]

Question 10 :

(i) Using properties of proportion, find $x : y$ if $\frac{x^3 + 12x}{6x^2 + 8} = \frac{y^3 + 27y}{9y^2 + 27}$ [3]

(ii) Draw a circle of radius 3 cm. Take a point P outside it. Without using the centre, draw two tangents to the circle from point P. [3]

(iii) Two poles AB and PQ are standing opposite each other on either side of a road 200 m wide. From a point R between them on the road, the angles of elevation of the top of the poles AB and PQ are 45° and 40° respectively. If height of AB = 80 m, find the height of PQ correct to the nearest metre. [4]

Solution :

(i) (a)

(ii) (a) Let $x - 1 = t$, then $t^2 - 5t - 6 = 0$

$\Rightarrow (t - 6)(t + 1) = 0 \Rightarrow t = 6$ or $-1 \Rightarrow x - 1 = 6$ or $x - 1 = -1 \Rightarrow x = 7$ or $x = 0$

(iii) (d) Since $(x - 2)$ is a factor of

$x^2 + 5x + p$, so, $2^2 + 5 \times 2 + p = 0 \Rightarrow p = -14$

(iv) (b) $r = \frac{1}{2} \times \left(-\frac{4}{3}\right) = -\frac{2}{3}$

(v) (c) Required term = $38 + (8 - 1) \times (-3) = 17$.

(vi) (d) $R_x (0, -1) = (0, 1)$

(vii) (b) Since $\frac{PB}{PD} = \frac{PC}{PE}$ and $\angle BPC = \angle DPE$, so, by SAS similarity axiom, $\Delta PBC \sim \Delta PDE$.

(viii) (a) After reshaping solids, their volumes remain the same.

(ix) (a) $-1 \leq 3 + 4x$ and $3 + 4x < 23, x \in R \Rightarrow -4 \leq 4x$ and $4x < 20, x \in R \Rightarrow -1 \leq x$ and $x < 5$

$\Rightarrow$ solution set is $\{-1 \leq x < 5, x \in R\}$

(x) (b) Since, there are 6 black face cards, so, required probability $= \frac{6}{52} = \frac{3}{26}$

(xi) (c) $MN = [1 \ -2] \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} = [2 + 2 \quad 1 - 4] = [4 \ -3]$

(xii) (c) Required coordinates are $\left(\frac{-1-3}{2}, \frac{4-2}{2}\right) = (-2, 1)$

(xiii) (b) $\angle AOC = 2 \times \angle ABC = 40°$

(xiv) (a)

(xv) (a) Mean of the given data $= \frac{2\times9+3\times4+5\times6+p\times3+9\times8}{9+4+6+3+8}$

$\Rightarrow 5 = \frac{132+3p}{30} \Rightarrow 132 + 3p = 150 \Rightarrow p = \frac{18}{3} = 6.$

Also, if $\bar{x} = \frac{x_1+x_2+...x_n}{n}$, then mean of $kx_1, kx_2, ... kx_n$ is $k\bar{x}$ not $k + \bar{x}$.

Question 2 :

(i) Amit deposits ₹1600 per month in a bank for 18 months in a recurring deposit account. If he gets ₹31,080 at the time of maturity, what is the rate of interest per annum? **[4]**

(ii) If $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b}$, prove that $ax + by + cz = 0$. **[4]**

(iii) A man invests ₹20,020 in buying shares of nominal value ₹26 at 10% premium. The dividend on the shares is 15% per annum. Calculate: **[4]**

(a) the number of shares he buys

(b) the dividend he receives annually

(c) the rate of interest he gets on his money.

Solution :

(i) We have, P = ₹1600, n = 18 months

Amount deposited in 18 months = ₹1,600 × 18 = ₹28,800

∴ S.I. = ₹(31,080 – 28,800) = ₹2,280

$$\text{S.I.} = P \times \frac{n(n+1)}{2} \times \frac{1}{12} \times \frac{R}{100} \Rightarrow 2280 = 1600 \times \frac{18 \times 19}{2} \times \frac{1}{12} \times \frac{R}{100} \Rightarrow R = \frac{2280}{4 \times 3 \times 19} = 10$$

Hence, rate of interest = 10% p.a. **Ans.**

(ii) We have, $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b} = k \Rightarrow x = k(b-c),\ y = k(c-a),\ z = k(a-b)$

Now, LHS = $ax + by + cz = ak(b-c) + bk(c-a) + ck(a-b)$

$= k(ab - ac + bc - ba + ca - bc) = k \times 0 = 0$ = **RHS** **Proved.**

(iii) (a) Market value of 1 share = 110% of ₹26 = ₹28.60.

Number of shares bought = $\frac{20020}{28.60}$ = 700 **Ans.**

(b) Face value of 700 shares = ₹26 × 700 = ₹18200

Dividend received = 15% of ₹18200 = ₹2730 **Ans.**

(c) Rate of return (or interest) = $\frac{2730 \times 100}{20020}$ % = 13.64% **Ans.**

Question 3 :

(i) A solid cylinder of radius 7 cm and height 14 cm is melted and recast into solid spheres each of radius 3.5 cm. Find the number of spheres formed. **[4]**

(ii) Find the equation of the line passing through (2, 4) and parallel to the line $3x + 5y - 15 = 0$. **[4]**

(iii) With the help of a graph paper, taking 1 cm = 1 unit along both x and y axis: **[5]**

(a) Plot points A (0, 3), B (2, 3), C (3, 0), D (2, –3), E (0, –3)

(b) Reflect points B, C and D on the y axis and name them as B', C' and D' respectively.

(c) Write the co-ordinates of B', C' and D'.

(d) Write the equation of line B'D'.

(e) Name the figure BCDD'C'B'B

Solution :

(i) We have, r = 7 cm, h = 14 cm, R = 3.5 cm

Let n spheres are formed.

∴ Volume of cylinder = n × volume of a sphere

$$\Rightarrow \pi r^2 h = n \times \frac{4}{3}\pi R^3 \Rightarrow 7 \times 7 \times 14 = n \times \frac{4}{3} \times (3.5)^3 \Rightarrow n = \frac{7 \times 7 \times 14 \times 3}{4 \times 3.5 \times 3.5 \times 3.5} = 12$$

Hence, 12 spheres are formed. **Ans.**

(ii) Let the equation of the line be $y = mx + c$ .... (1)

∵ (1) passes through (2, 4)

∴ $4 = m \times 2 + c \Rightarrow 2m + c = 4$ ...(2)

Also, (1) is parallel to the line $3x + 5y = 15$ or $y = -\frac{3}{5}x + 3$

∴ Gradient m of (1) $= -\frac{3}{5}$

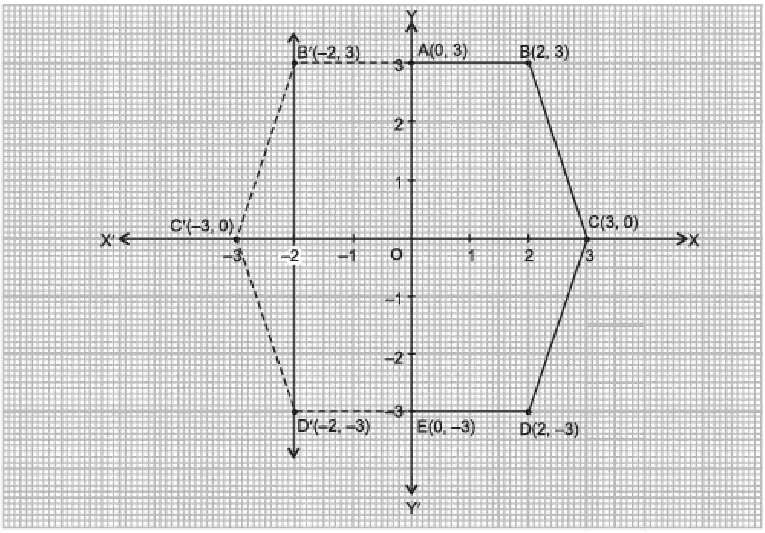
Substituting $m = -\frac{3}{5}$ in (2), we get $2 \times \left(-\frac{3}{5}\right) + c = 4 \Rightarrow c = 4 + \frac{6}{5} = \frac{26}{5}$

Substituting $m = -\frac{3}{5}$ and $c = \frac{26}{5}$ in (1), we get

$y = -\frac{3}{5}x + \frac{26}{5} \Rightarrow 5y = -3x + 26 \Rightarrow 3x + 5y = 26$ **Ans.**

(iii) (a) The given points have been plotted on the graph paper as shown below.

(b) The points B′, C′ and D′ have been shown on the graph paper.



(c) The coordinates of the points B′, C′ and D′ are respectively (–2, 3), (–3, 0) and (–2, –3) **Ans.**

(d) Equation of the line B′D′ is $x = -2$. **Ans.**

(e) The figure BCDD′C′B′B is a hexagon. **Ans.**

Question 4 :

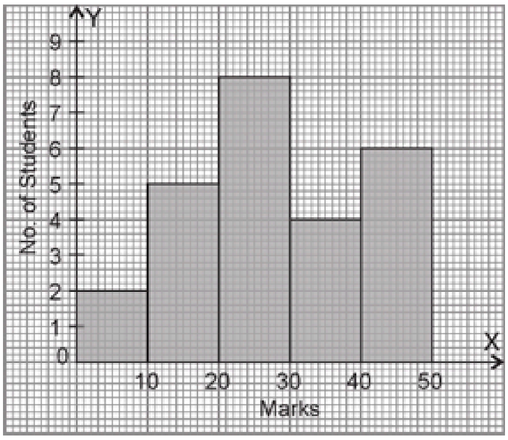
(i) A shopkeeper bought an article with marked price ₹1200 from the wholesaler at a discount of 10%. The shopkeeper sells this article to the customer on the marked price printed on it. If the rate of GST is 6%, then find : [3]

(a) GST paid by the wholesaler.

(b) Amount paid by the customer to buy the item.

(ii) Find the sum of first 8 terms of the GP $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots$ [3]

(iii) The histogram represents the scores obtained by 25 students in a Mathematics mental test. Use the data to:



(a) Frame a frequency distribution table.

(b) To calculate mean.

(c) To determine the modal class. [4]

Solution :

(i) SP for wholesaler = 90% of ₹1200 = $\frac{90}{100}$ × ₹1200 = ₹1080

Rate of GST = 6%

(a) ∴ GST paid by the wholesaler to the government = 6% of ₹1080

= $\frac{6}{100}$ × ₹1080 = ₹64.80 **Ans.**

(b) Price at which the shopkeeper sold the article = ₹1200

Rate of GST = 6%

∴ Amount paid by the customer = ₹1200 + 6% of ₹1200 = 106% of ₹1200 = $\frac{106}{100}$ × ₹1200 = ₹1272 **Ans.**

(ii) $a = \frac{1}{3}, r = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2} < 1$

So, $S_8 = \frac{a(1-r^8)}{(1-r)} = \frac{\frac{1}{3}\left[1-\left(\frac{1}{2}\right)^8\right]}{1-\frac{1}{2}} = \frac{2}{3} \times \left(1 - \frac{1}{256}\right) = \frac{2}{3} \times \frac{255}{256} = \frac{85}{128}$ **Ans.**

(iii) (a) The frequency distribution table may be prepared as under :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	2	5	8	4	6

(b)

Marks	Mid-value (x)	No. of students (f)	fx
0 – 10	5	2	10
10 – 20	15	5	75
20 – 30	25	8	200
30 – 40	35	4	140
40 – 50	45	6	270
		$\Sigma f = 25$	$\Sigma fx = 695$

Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{695}{25} = 27.8$ **Ans.**

(c) Clearly the modal class is 20 – 30 **Ans.**

Question 5 :

(i) Given $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$ **[3]**

Write down

(a) the order of the matrix X **(b) the matrix X.**

(ii) In the map of a rectangular plot of land, the length = 2.5 cm and breadth = 1.4 cm. If the scale is 1:10000, find the area of the plot in m^2. **[3]**

(iii) Find the value of' 'k' if $4x^3 - 2x^2 + kx + 5$ leaves remainder –10 when divided by $2x + 1$. **[4]**

Solution :

(i) (a) Let A = $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix}$. Then, AX = $\begin{bmatrix} 12 \\ 10 \end{bmatrix}$

Since AX exists, we have

Number of rows in X = Number of columns in A = 2

Number of columns in X = Number of columns in AX = 1.

Thus, X has 2 rows and 1 column.

$\therefore$ The order of X is (2 × 1). **Ans.**

(b) Let, $X = \begin{bmatrix} a \\ b \end{bmatrix}$. Then

$$\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 8a & -2b \\ a & +4b \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

$\Rightarrow$ $8a - 2b = 12$ and $a + 4b = 10$

Solving the above two equations, we get, $a = 2$ and $b = 2$

Hence, $X = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. **Ans.**

(ii) Seale factor, $k = \frac{1}{10000}$

Area of the plot $= \frac{1}{k^2} \times$ area in the map

$= (10000)^2 \times 2.5 \times 1.4 \text{ cm}^2 = \frac{(10000)^2 \times 2.5 \times 1.4}{10000} \text{ m}^2 = 35{,}000 \text{ m}^2$ **Ans.**

(iii) Let $f(x) = 4x^3 - 2x^2 + kx + 5$

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

On dividing $f(x)$ by $(2x + 1)$, we get the remainder as $f\left(-\frac{1}{2}\right)$.

$$\therefore \quad f\left(-\frac{1}{2}\right) = 4 \times \left(-\frac{1}{2}\right)^3 - 2 \times \left(-\frac{1}{2}\right)^2 + k \times \left(-\frac{1}{2}\right) + 5$$

$$= 4 \times \left(-\frac{1}{8}\right) - 2 \times \frac{1}{4} - \frac{k}{2} + 5 = -\frac{1}{2} - \frac{1}{2} - \frac{k}{2} + 5 = 4 - \frac{k}{2}$$

But, $f\left(-\frac{1}{2}\right) = -10$ [Given] $\Rightarrow -10 = 4 - \frac{k}{2} \Rightarrow \frac{k}{2} = 14 \Rightarrow k = 28$ **Ans.**

Question 6 :

(i) Determine whether the line through (–2, 3) and (4, 1) is perpendicular to the line $3x = y + 1$. Does the line $3x = y + 1$ bisect the line joining (–2, 3) and (4, 1)? [3]

(ii) Show that $\frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\sec A + 1}{\sec A - 1}$. [3]

(iii) The first term of an AP is 2 and the last term is 50. If the sum of all these terms is 442, find its common difference. [4]

Solution :

(i) Gradient (m_1) of the line through (–2, 3) and (4, 1) = $\frac{1-3}{4+2} = \frac{-2}{6} = \frac{-1}{3}$

Gradient (m_2) of the line $3x = y + 1$ is 3

$\because$ $m_1 \times m_2 = -1$, hence the lines are perpendicular.

Coordinates of the mid-point of the line joining (–2, 3) and (4, 1) are $\left(\frac{4-2}{2}, \frac{1+3}{2}\right) = (1, 2)$

Substituting $x = 1$ and $y = 2$ in $3x = y + 1$, we have, $3 = 1 + 2 = 3$

Hence, the line $3x = y + 1$ bisects the line joining (–2, 3) and (4, 1) **Ans.**

(ii) We have,

$$\frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\sec A + 1}{\sec A - 1}$$

$$\text{L.H.S.} = \frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A} = \frac{\sin A\left(\frac{1}{\cos A} + 1\right)}{\sin A\left(\frac{1}{\cos A} - 1\right)} = \frac{\sec A + 1}{\sec A - 1} = \textbf{R.H.S} \quad \textbf{Proved.}$$

(iii) Here, $a = 2$, $l = 50$ and $S_n = 442$.

Now, $S_n = \frac{n}{2}(a + l)$

$\Rightarrow \quad 442 = \frac{n}{2}(2 + 50) \Rightarrow n = \frac{442 \times 2}{52} = 17$

Now, $l = a + (n - 1)d$

$\Rightarrow 50 = 2 + (17 - 1) \times d \Rightarrow 48 = 16d \Rightarrow d = 3$

Hence, common difference of the AP is 3. **Ans.**

Question 7 :

(i) An aeroplane travelled a distance of 400 km at an average speed of x km/h. On return journey, the speed was increased by 40 km/h. Write down an expression for the time taken for

(a) the onward journey (b) the return journey.

If the return journey took 30 minutes less than the onward journey, write down an equation in x and find its value [5]

(ii) The following table shows the distribution of marks in Mathematics:

Marks (less than)	No. of students
10	7
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With the help of a graph paper, taking 2 cm = 10 units along one axis and 2 cm = 10 units along the other axis, draw an ogive for the above distribution and use it to find the:

(a) median.

(b) number of students who scored distinction marks (75% and above)

(c) number of students, who passed the examination if pass marks is 35%. [5]

Solution :

(i) (a) Distance covered = 400 km

Speed = x km/h

$\therefore$ Time taken = $\frac{400}{x}$ hours **Ans.**

(b) Distance covered = 400 km; Speed = $(x + 40)$ km/h

$\therefore$ Time taken = $\frac{400}{x+40}$ hours **Ans.**

We have, $\frac{400}{x} - \frac{400}{x+40} = \frac{30}{60} \Rightarrow \frac{x+40-x}{x(x+40)} = \frac{1}{2 \times 400} \Rightarrow x^2 + 40x = 32000$ **Ans.**

$\Rightarrow x^2 + 200x - 160x - 32000 = 0 \Rightarrow x(x + 200) - 160(x + 200) = 0$

$\Rightarrow (x + 200)(x - 160) = 0 \Rightarrow x = -200$ or $x = 160$

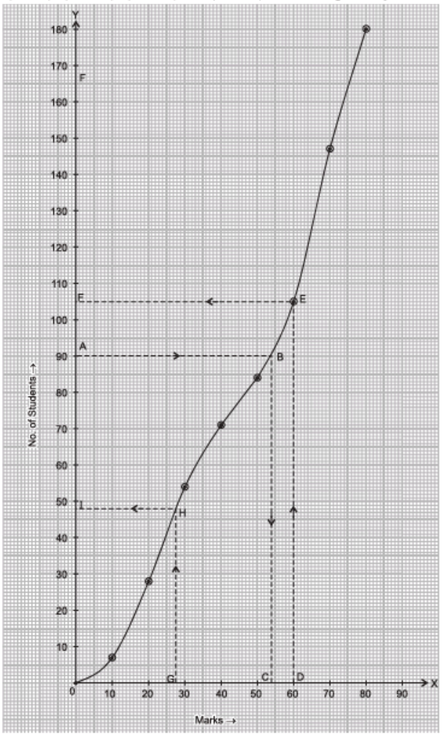
Hence, $x = 160$ [Rejecting –ve speed] **Ans.**

(ii) The cumulative frequency table may be prepared as below :

Marks	Number of students	Cumulative frequency
0–10	7	7
10–20	21	28
20–30	26	54
30–40	17	71

40–50	13	84
50–60	21	105
60–70	42	147
70–80	33	180

Now, we take marks along x-axis and number of students along y-axis. We plot the points (0, 0), (10, 7), (20, 28), (30, 54), (40, 71), (50, 84), (60, 105), (70, 147) and (80, 180). Join these points by a free hand curve to get the ogive.



(a) Here N = 180 ⇒ $\frac{N}{2} = 90$

On the graph paper take point A on the y-axis, representing 90. Through A, draw a horizontal line meeting the ogive at B. From B, draw BC ⊥ x-axis, meeting the x-axis at C. The abscissa of C is 54. Hence, median marks = 54. **Ans.**

(b) Maximum marks = 80

∴ 75% of 80 = $\frac{80 \times 75}{100} = 60$

From the ogive, we see that number of students who scored less than 75% marks (60 marks) = 105

∴ No. of students who scored 75% and above = 180 – 105 = 75 **Ans.**

(c) 35% of 80 = $\frac{80 \times 35}{100} = 28$

From the ogive, we see that number of students who scored less than 35% marks (28 marks) = 48

∴ No. of students who passed the examination = 180 – 48 = 132 **Ans.**

Question 8 :

(i) A box consists of 4 red, 5 black and 6 white balls. One ball is drawn out at random. Find the probability that the ball drawn is: [3]

(a) black

(b) red or white

(ii) Draw a line segment AB = 3 cm. Draw the locus of a point P which moves at a distance of 5 cm from AB. [3]

(iii) In the figure, O is the centre of the circumcircle of ∠XYZ. Tangents at X and Y intersect at T. Given ∠XTY = 80° and ∠XOZ = 140°, calculate the value of ∠ZXY. [4]

Solution :

(i) Total no. of balls = 4 + 5 + 6 = 15

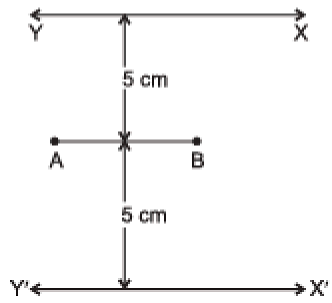
(a) No. of black balls = 5

$$P\text{ (a black ball)} = \frac{\text{no. of black balls}}{\text{total no. of balls}} = \frac{5}{15} = \frac{1}{3}$$ **Ans.**

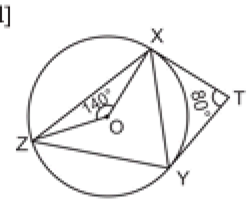
(b) No. of red or white balls = 4 + 6 = 10

$$P\text{ (a red or a white ball)} = \frac{\text{no. of red or white balls}}{\text{total no. of balls}} = \frac{10}{15} = \frac{2}{3}$$ **Ans.**

(ii)



(iii) TX and TY are tangents from T



$\therefore$ TX = TY [Tangents drawn from an exterior point to the circle are equal]

$\Rightarrow$ $\angle TYX = \angle TXY$ [Angles opposite to equal sides are equal]

$\Rightarrow$ $\angle TYX = \angle TXY = \dfrac{180° - 80°}{2} = 50°$

Also, $\angle TXY = \angle XZY$ [Angles in the alternate segments]

$\Rightarrow$ $\angle XZY = 50°$

Now, $\angle XYZ = \dfrac{1}{2}\angle XOZ$ [Angle at the centre is double the angle at the circumference]

$\Rightarrow$ $\angle XYZ = \dfrac{1}{2} \times 140° = 70°$

In ΔXYZ, $\angle ZXY = 180° - (\angle XYZ + \angle XZY)$ [Angle sum property of a Δ]

$\Rightarrow \angle ZXY = 180° - (70° + 50°) = 180° - 120° = 60°$ **Ans.**

Question 9 :

(i) Solve the inequation $2x - \frac{5}{2} < x + \frac{3}{2} \le 3x + \frac{11}{2}$, $x \in I$. Represent the solution set on a number line. **[3]**

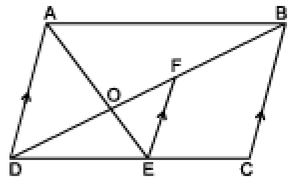
(ii) The mean of the following distribution is 49. Find the missing frequency *a*. **[3]**

Class	**0 – 20**	**20 – 40**	**40 – 60**	**60 – 80**	**80 – 100**
Frequency	**15**	**20**	**30**	***a***	**10**

(iii) In the figure, ABCD is a parallelogram. E is the point on CD. AE intersects BD at O. EF ∥ CB. Find the following ratios, if DF : FB = 3 : 1. **[4]**

(a) ar(ΔDFE) : ar (ΔDBC) **(b) ar(ΔOFE) : ar (ΔODA)**

(c) ar(ΔDFE) : ar (trap.EFBC)



Solution :

(i) We have, $2x - \frac{5}{2} < x + \frac{3}{2} \le 3x + \frac{11}{2}$, $x \in I$

$\Rightarrow \quad 2x - \frac{5}{2} < x + \frac{3}{2}$ and $x + \frac{3}{2} \le 3x + \frac{11}{2}$, $x \in I$

$\Rightarrow \quad x < \frac{3}{2} + \frac{5}{2}$ and $-2x \le \frac{11}{2} - \frac{3}{2}$, $x \in I \Rightarrow x < 4$ and $x \ge -2$, $x \in I$

$\therefore$ Solution set $= \{x : -2 \le x < 4, x \in I\} = \{-2, -1, 0, 1, 2, 3\}$

The graph of the solution set is shown below :

(ii) We may prepare a table as below :

Class	Mid value (x)	Frequency (f)	$f \times x$
0 – 20	10	15	150
20 – 40	30	20	600
40 – 60	50	30	1500
60 – 80	70	a	$70a$
80 – 100	90	10	900
		$\Sigma f = 75 + a$	$\Sigma fx = 3150 + 70a$

Now, Mean $= \frac{\Sigma fx}{\Sigma f}$

$\Rightarrow 49 = \frac{3150 + 70a}{75 + a} \Rightarrow 3675 + 49a = 3150 + 70a \Rightarrow 21a = 3675 - 3150 \Rightarrow a = \frac{525}{21} = 25$ **Ans.**

Now, Mean $= \frac{\Sigma fx}{\Sigma f}$

$\Rightarrow 49 = \frac{3150 + 70a}{75 + a} \Rightarrow 3675 + 49a = 3150 + 70a \Rightarrow 21a = 3675 - 3150 \Rightarrow a = \frac{525}{21} = 25$ **Ans.**

(iii) (a) ΔDEF ~ ΔDCB [AA similarity]

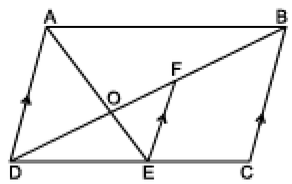
$\Rightarrow \frac{DF^2}{DB^2} = \frac{\text{area }(\Delta DEF)}{\text{area }(\Delta DBC)}$

= Now, $\frac{FB}{DF} = \frac{1}{3}$ [Given]

$\Rightarrow \frac{FB}{DF} + 1 = \frac{1}{3} + 1 \Rightarrow \frac{FB + DF}{DF} = \frac{1+3}{3} \Rightarrow \frac{DB}{DF} = \frac{4}{3} \Rightarrow \frac{DF}{DB} = \frac{3}{4}$

Hence, $\frac{\text{area }(\Delta DFE)}{\text{area }(\Delta DBC)} = \frac{DF^2}{DB^2} = \frac{9}{16}$ **Ans.**

(b) $\frac{\text{area }(\Delta OFE)}{\text{area }(\Delta ODA)} = \frac{EF^2}{AD^2}$ [∵ ΔOFE ~ ΔODA]



Now, $\frac{EF}{AD} = \frac{EF}{BC}$ [∵ AD = BC, opp. sides of ∥ gm]

$= \frac{DF}{DB} = \frac{3}{4}$ [ΔDFE ~ ΔDBC]

$\Rightarrow \frac{\text{area }(\Delta OFE)}{\text{area }(\Delta ODA)} = \frac{3^2}{4^2} = \frac{9}{16}$ **Ans.**

(c) $\frac{\text{area }(\Delta DFE)}{\text{area }(\Delta DBC)} = \frac{9}{16}$ [From (a)]

$\Rightarrow \frac{\text{area }(\Delta DFE)}{\text{area }(\Delta DBC) - \text{area }(\Delta DFE)} = \frac{9}{16 - 9}$ [Dividendo and invertendo]

$= \frac{\text{area }\Delta DFE}{\text{area trap. EFBC}} = \frac{9}{7}$ **Ans.**

Question 10 :

(i) **Using properties of proportion, find $x : y$ if $\dfrac{x^3 + 12x}{6x^2 + 8} = \dfrac{y^3 + 27y}{9y^2 + 27}$** **[3]**

(ii) **Draw a circle of radius 3 cm. Take a point P outside it. Without using the centre, draw two tangents to the circle from point P.** **[3]**

(iii) **Two poles AB and PQ are standing opposite each other on either side of a road 200 m wide. From a point R between them on the road, the angles of elevation of the top of the poles AB and PQ are 45° and 40° respectively. If height of AB = 80 m, find the height of PQ correct to the nearest metre.** **[4]**

Solution :

(i) We have, $\dfrac{x^3 + 12x}{6x^2 + 8} = \dfrac{y^3 + 27y}{9y^2 + 27}$

Applying componendo and dividendo, we get

$$\frac{x^3 + 12x + 6x^2 + 8}{x^3 + 12x - 6x^2 - 8} = \frac{y^3 + 27y + 9y^2 + 27}{y^3 + 27y - 9y^2 - 27}$$

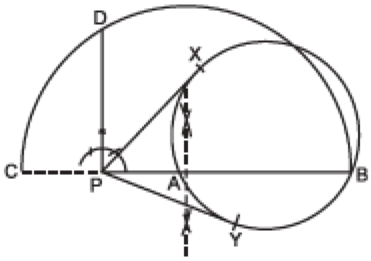
$$\Rightarrow \frac{x^3 + 2^3 + 3 \times x \times 2(x+2)}{x^3 - 2^3 - 3 \times x \times 2(x-2)} = \frac{y^3 + 3^3 + 3 \times y \times 3(y+3)}{y^3 - 3^3 - 3 \times y \times 3(y-3)}$$

$$\Rightarrow \frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$ [$\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)$ and $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$]

$$\Rightarrow \frac{x+2}{x-2} = \frac{y+3}{y-3} \Rightarrow \frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$ [Applying componendo and dividendo]

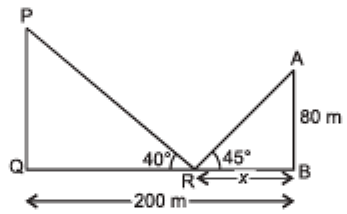
$$\Rightarrow \frac{2x}{4} = \frac{2y}{6} \Rightarrow \frac{x}{2} = \frac{y}{3} \Rightarrow \frac{x}{y} = \frac{2}{3} \Rightarrow x : y = 2 : 3$$ **Ans.**

(ii) **Steps of construction :**



1. Draw a circle of radius 3 cm. Take any point P, outside the circle.
2. Draw secant PAB, intersecting the circle at A and B.
3. Extend AP to C to such that AP = PC.
4. Taking BC as diameter, draw a semicircle.
5. At P, draw PD ⊥ BC, which cuts the semicircle at D.
6. With P as centre and radius equal to PD, draw two arcs, which cut the circle at X and Y.
7. Join PX and PY to get the required tangents.

(iii) Let BR = x m.



Then in ΔABR,

$\tan 45° = \dfrac{AB}{BR} \Rightarrow 1 = \dfrac{80}{x} \Rightarrow x = 80$ m

$\therefore$ QR = QB – BR = (200 – 80) m = 120 m

In ΔPQR, $\tan 40° = \dfrac{PQ}{QR} \Rightarrow 0.8390 = \dfrac{PQ}{120} \Rightarrow PQ = 120 \times 0.8390 = 100.68$ m

Hence, height of PQ = 101 m **Ans.**