Subject Code: YC MATHS 8	YASHWANT CLASSES Estd.1995	Subject: MATHS

Answer to this Paper must be written on the paper provided separately.
You will not be allowed to write during first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.
Section A is compulsory. Attempt any four questions from Section B.
The intended marks for questions or parts of questions are given in brackets [].

SECTION A (40 MARKS)

(Attempt all questions from this Section)

1. **Question 1 Choose the correct answers to the questions from the given options:** **[15]**

(a) A retailer purchases a fan for ₹1500 from a wholesaler and sells it to a consumer at 10% profit. If the sales are intra-state and the rate of GST is 12%, the cost of the fan to the consumer inclusive of tax is: **[1]**

a) ₹1848 b) ₹1830

c) ₹1650 d) ₹1800

(b) A factory kept increasing its output by the same percentage every year. Then, the percentage, if it is known that the output is doubled in the last two years, will be **[1]**

a) 44.4% b) 14.4%

c) 41.4% d) 44.1%

(c) When $ax^3 + 6x^2 + 4x + 5$ is divided by $(x + 3)$, the remainder is -7. The value of constant a is **[1]**

a) 2 b) -2

c) -3 d) 3

(d) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then the value of matrix A^5 is **[1]**

a) $\begin{bmatrix} 87 & 149 \\ 149 & -62 \end{bmatrix}$ b) $\begin{bmatrix} 87 & 149 \\ 149 & 62 \end{bmatrix}$

c) $\begin{bmatrix} 62 & 149 \\ -149 & -87 \end{bmatrix}$ d) $\begin{bmatrix} -62 & -149 \\ 149 & 87 \end{bmatrix}$

(e) An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is **[1]**

a) 3 b) 6

c) 5 d) 2

(f) If (4, 3) and (-4, -3) are opposite two vertices of a rectangle, then other two vertices are **[1]**

a) (4, -3) and (-4, 3) b) (-4, -3) and (-4, -3)

c) (-4, 4) and (-3, 4) d) (4, -3) and (-3, 4)

(g) Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC at L and AD produced at E. The values of EL and ar (△AEL) are respectively **[1]**

a) ar (△CBL) and BL b) 2BL and 4 ar (△CBL)

c) 4 ar (△CBL) and 2BL d) BL and ar (△CBL)

(h) A sphere of radius a units is immersed completely in water contained in a right circular cone of semi-vertical angle 30° and water is drained off from the cone till its surface touches the sphere. Then, the volume of water remaining in the cone will be **[1]**

a) $\frac{5}{3}\pi a^2$ b) $\frac{5\pi}{3}a^3$

c) $\frac{\pi a^3}{3}$ d) $5\pi a^3$

(i) Graph the range of the inequation $-2\frac{2}{3} \leq x + \frac{1}{3} \leq 3\frac{1}{3}, \forall x \in R$ on the number line. If the solution set is consider as a diagonal of a square on the number line, then the area of obtained figure, is **[1]**

a) 11 sq units b) 14 sq units

c) 17 sq units d) 18 sq units

(j) The probability that the minute hand lies from 5 to 15 min in the wall clock, is **[1]**

a) $\frac{1}{6}$ b) $\frac{5}{6}$

c) $\frac{1}{5}$ d) $\frac{1}{10}$

(k) If $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then A^n (where, n is a natural number) is equal to **[1]**

a) $\begin{bmatrix} 3n & 0 \\ 0 & 3n \end{bmatrix}$ b) $3^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ d) $I_{2 \times 2}$

(l) The sum of the squares of the distances of a moving point (x, y) from two fixed points (a, 0) and (- a, 0) is equal to a constant quantity $2b^2$. The value of $x^2 + y^2 + a^2$ is equal to **[1]**

a) b^2

b) $-a^2$

c) ab

d) $-b^2$

(m) If P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. Then, the perimeter of the quadrilateral PQSR will be **[1]**

a) $2(\sqrt{3} + 1)r$

b) $2\sqrt{3} + r$

c) 2r

d) $2\sqrt{3}r$

(n) Observe the data given in three sets **[1]**

P: 3, 5, 9, 12, x, 7, 2

Q: 8, 2, 1, 5, 7, 9, 3

R: 5, 9, 8, 3, 2, 7, 1

If the ratio between P's and Q's means is 7 : 5, then the ratio between P's and R's means is

a) 7 : 5

b) 5 : 7

c) 6 : 7

d) 7 : 6

(o) **Assertion (A):** Sum of first 10 terms of the arithmetic progression -0.5, -1.0, -1.5, ... is 27.5 **[1]**

Reason (R): Sum of n terms of an A.P. is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a = first term, d = common difference.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

2. **Question 2** **[12]**

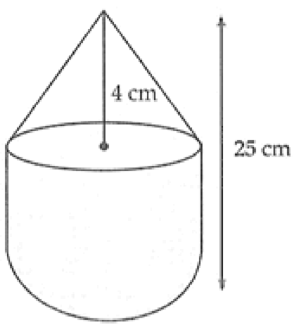
(a) Mrs. chopra deposits ₹1600 per month in a Recurring Deposit Account at 9% per annum simple interest. If she gets ₹65592 at the time of maturity, then find the total time for which the account was held. **[4]**

(b) Find the mean proportional of $(a^4 - b^4)^2$ and $[(a^2 - b^2)(a - b)]^{-2}$. **[4]**

(c) If cosec $\theta = x + \frac{1}{4x}$, then prove that cosec θ + cot $\theta = 2x$ or $\frac{1}{2x}$. **[4]**

3. **Question 3** **[13]**

(a) The given solid figure is cylinder surmounted by a cone. The diameter of the base of the cylinder is 6 cm. The height of the cone is 4 cm and the total height of the solid is 25 cm. Take $\pi = \frac{22}{7}$. **[4]**



Find the:

i. Volume of the solid

ii. Curved surface area of the solid

Give your answer correct to the nearest whole number.

(b) The equation of a line is y = 3x - 5. Write down the slope of this line and the intercept made by its on the Y-axis. Hence or otherwise, write down the equation of a line, which is parallel to the line and which passes through the point (0, 5). **[4]**

(c) Use graph paper for this question (Take 2 cm = 1 unit along both x and y axis). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, - 2), C (0, -1) and D (0, 1) **[5]**

i. Reflect quadrilateral ABCD on the y-axis and name it as A'B'CD.

ii. Write down the coordinates of A' and B'

iii. Name two points which are invariant under the above reflection.

iv. Name the polygon A'B'CD.

Section B

Attempt any 4 questions

4. **Question 4** **[10]**

(a) The price of a Barbie Doll is ₹ 3136 inclusive tax (under GST) at the rate of 12% on its listed price. A buyer asks for a discount on the listed price, so that after charging GST, the selling price becomes equal to the listed price. Find the amount of discount which the seller has to allow for the deal. **[3]**

(b) Find the values of k, for which the equation $x^2 + 5kx + 16 = 0$ has no real roots. **[3]**

(c) The mean of the following distribution is 49. Find the missing frequency **a**. **[4]**

Class Interval	0-20	20-40	40-60	60-80	80 -100
Frequency	15	20	30	a	10

5. **Question 5** **[10]**

(a) Find the values of x, y, a and b, when $\begin{bmatrix} x+y & a-b \\ a+b & 2x-3y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -1 & -5 \end{bmatrix}$. **[3]**

(b) Two chords AB and CD of a circle intersect each other at a point E inside the circle. If AB = 9 cm, AE = 4 cm and ED = 6 cm, then find CE. **[3]**

(c) Determine, whether the polynomial g(x) = x - 7 is a factor of $f(x) = x^3 - 6x^2 - 19x + 84$ or not. **[4]**

6. **Question 6** **[10]**

(a) Find the points of trisection of the line segment joining the points (5, -6) and (-7, 5). **[3]**

(b) Prove the following identities. **[3]**

i. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

ii. $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

(c) 150 workers were engaged to finish a job in a certain number of days, 4 workers dropped out on second day, 4 more workers dropped out an third day and so on. It took 8 more days of finish the work. Find the number of days in which the work was completed. **[4]**

7. **Question 7** **[10]**

(a) A two-digit positive number, such that the product of its digits is 6. If 9 is added to the number, then the digits interchange their places. Find the number. **[5]**

(b) The marks obtained by 120 students in a test are given below: **[5]**

Marks	Number of Students
0 - 10	5
10 - 20	9
20 - 30	16
30 - 40	22
40 - 50	26
50 - 60	18
60 - 70	11
70 - 80	6
80 - 90	4
90 - 100	3

Draw an ogive for the given distribution on a graph sheet.

(Use suitable scale for ogive to estimate the following)

i. the median.

ii. the number of students who obtained more than 75% marks in the test.

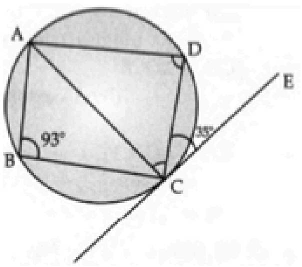
iii. the number of students who did not pass the test, if minimum marks required to pass is 40.

8. **Question 8** **[10]**

(a) Two players Niharika and Shreya play a tennis match. It is known that the probability of Niharika winning the match is 0.62. What is the probability of Shreya winning the match? **[3]**

(b) A conical military tent is 5 m high and the diameter of the base is 24 m. Find the cost of canvas used in making this tent at the rate of ₹ 14 per sq m. **[3]**

(c) In the given figure CE is a tangent to the circle at point C. ABCD is a cyclic quadrilateral. If $\angle ABC = 93^{0}$ and $\angle DCE = 35^{0}$ **[4]**



find:

i. $\angle ADC$

ii. $\angle CAD$

iii. $\angle ACD$

9. **Question 9** **[10]**

(a) Given: $A = \{x : 3 < 2x - 1 < 9, x \in R\}$, $B = \{x : 11 \leq 3x + 2 \leq 23, x \in R\}$ where R is the set of real number. **[3]**

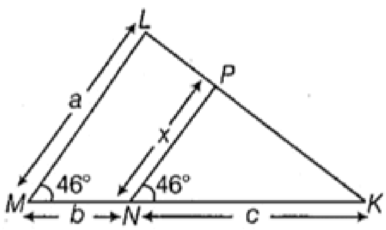
i. Represents A and B on number lines

ii. On the number line also mark $A \cap B$.

(b) Find the missing frequency for the given frequency distribution table, if the mean, of the distribution is 18. **[3]**

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

(c) In the given figure, $\angle M = \angle N = 46^o$. Express x in terms of a, b and c, where a, b and c are the lengths of LM, MN and NK, respectively. **[4]**

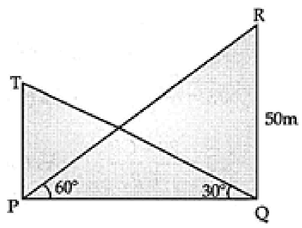


10. **Question 10** **[10]**

(a) The ages of A and B are in the ratio 7 : 8. Six years ago, their ages were in the ratio 5 : 6. Find their present ages. **[3]**

(b) Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. **[3]**

(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60^o and that the tower PT from a point Q is 30^o. Find the height to the tower PT, correct to the nearest metre. **[4]**



Section A

1. Question 1 Choose the correct answers to the questions from the given options:

(i) **(a)** ₹1848

Explanation: {

Here, selling price of fan = ₹1650

GST on fan = 12% of ₹ 1650

= $1650 \times \frac{12}{100}$

= 198

Thus, cost of a fan to the consumer inclusive of tax

= ₹(1650 + 198) = ₹1848

(ii) **(c)** 41.4%

Explanation: {

Let P be the initial production (2 yr ago) and the increase in production every year be x%. Then, production at the end of first year = $P + \frac{Px}{100} = P\left(1 + \frac{x}{100}\right)$

Production at the end of second year

$= P = \left(1 + \frac{x}{100}\right) + \frac{Px}{100}\left[\left(1 + \frac{x}{100}\right)\right]$

$= P\left(1 + \frac{x}{100}\right)\left(1 + \frac{x}{100}\right) = P\left(1 + \frac{x}{100}\right)^2$

Since, the production is doubled in last two years.

$\therefore P\left(1 + \frac{x}{100}\right)^2 = 2P \Rightarrow \left(1 + \frac{x}{100}\right)^2 = 2$

$\Rightarrow (100 + x)^2 = 2 \times (100)^2 \Rightarrow 10000 + x^2 + 200x = 20000$

$\Rightarrow$ On comparing it with $ax^2 + bx + c = 0$, we get

a = 1, b = 200 and c = -10000

By quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-200 \pm \sqrt{(200)^2 + 40000}}{2}$

$= -100 \pm 100\sqrt{2} = 100(-1 \pm 4\sqrt{2})$

= 100 (-1 + 1.414) [∵ percentage cannot be negative]

= 100(0.414) = 41.4

Hence, the required percentage is 41.4%.

(iii) **(a)** 2

Explanation: {

Let $f(x) = ax^3 + 6x^2 + 4x + 5$

By remainder theorem, f(-3) = -7

$\Rightarrow a(-3)^3 + 6(-3)^2 + 4(-3) + 5 = -7$

$\Rightarrow -27a + 54 - 12 + 5 = -7$

$\Rightarrow -27a = -54$

$\Rightarrow a = 2$

(iv) **(c)** $\begin{bmatrix} 62 & 149 \\ -149 & -87 \end{bmatrix}$

Explanation: {

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\therefore A^4 = A^2 \cdot A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 64-25 & 40+15 \\ -40-15 & -25+9 \end{bmatrix}$

$= \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

Now, $A^5 = A^4 \cdot A = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 117-55 & 39+110 \\ -165+16 & -55-32 \end{bmatrix} = \begin{bmatrix} 62 & 149 \\ -149 & -87 \end{bmatrix}$

(v) **(d)** 2

Explanation: {

Given, $S_{11} = 33$

$\rightarrow \frac{11}{2}(2a + 10d) = 33$ [$\because S_n = \frac{n}{2}[2a + (n-1)d]$]

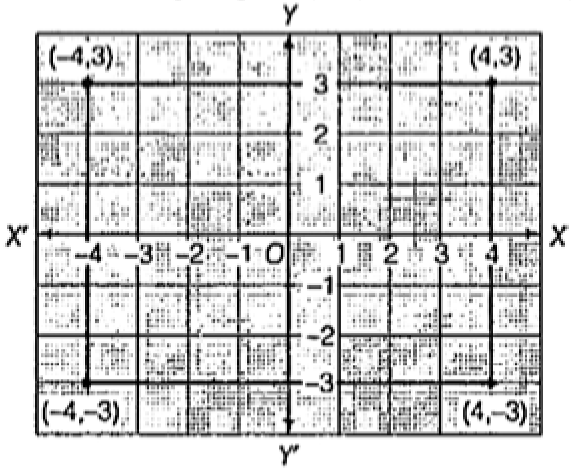
$\Rightarrow a + 5d = 3$

i.e. $a_6 = 3 \Rightarrow a_4 = 2$ [$\because$ alternate terms are integers and the given sum is possible]

(vi) **(a)** (4, -3) and (-4, 3)

Explanation: {

Since, the image of point (4, 3) under X-axis is (4, - 3) and the image of point (4, 3) under Y-axis is (-4, 3).

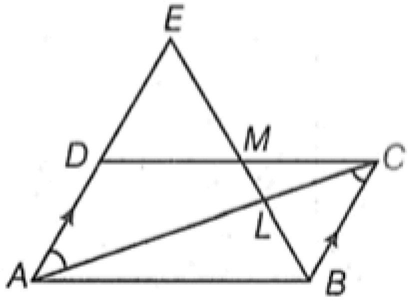


$\therefore$ Other two vertices of the rectangle are (4, -3) and (-4, 3).

(vii) **(b)** 2BL and 4 ar (△CBL)

Explanation: {

In △BMC and △EMD, we have



∠BMC = ∠EMD [vertically opposite angles]

⇒ MC = MD [∵ M is the mid-point of CD]

⇒ ∠MCB = ∠MDE [alternate angles]

So, by AAS congruence criterion, we have

$\triangle BMC \cong \triangle EMD$

⇒ BC = ED [∵ corresponding parts of congruent triangles are equal]

In △AEL and △CBL, we have

∠ALE = ∠CLB [vertically opposite angles]

and ∠EAL = ∠BCL [alternate angles]

So, by AA criterion of similarity, we have

$\triangle AEL \sim \triangle CBL$

$\Rightarrow \frac{AE}{BC} = \frac{EL}{BL} = \frac{AL}{CL}$ [∵ if two triangles are similar, then their corresponding sides are proportional]

On taking first two terms, we get

$\frac{EL}{BL} = \frac{AE}{BC} = \frac{AD+DE}{BC}$

$= \frac{BC+BC}{BC} = \frac{2BC}{BC}$ = 2 [∵ AD = SC as sides opposite to parallelogram and DE = BC, proved above]

⇒ EL = 2BL ...(i)

Now, $\frac{\text{ar}(\Delta AEL)}{\text{ar}(\Delta CBL)} = \left(\frac{EL}{BL}\right)^2$ [∵ ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides]

$= \left(\frac{2BL}{BL}\right)^2$ = $(2)^2$ [from Eq. (i)]

$\Rightarrow \frac{\text{ar}(\triangle AEL)}{\text{ar}(\triangle CBL)}$ = 4

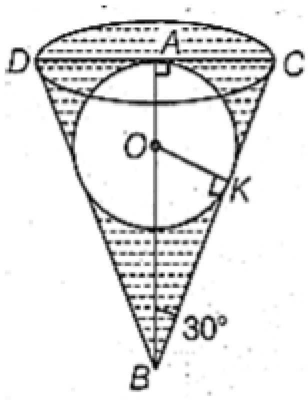
⇒ ar (△AEL) = 4 ar (△CBL)

(viii) **(b)** $\frac{5\pi}{3}a^3$

Explanation: {

Let radius of sphere be a, i.e. OK = OA = a.

Then, the centre O of a sphere will be centroid of the $\triangle$BCD



$\therefore$ OA = $\frac{1}{3}$AB $\Rightarrow$ AB = 3(OA)

In right angled $\triangle$OKB,

sin $30^0 = \frac{OK}{OB} = \frac{a}{OB}$

$\Rightarrow \frac{1}{2} = \frac{a}{OB}$

$\Rightarrow$ OB = 2a

Now, AB = OA + OB = a + 2a = 3a

Now, in right angled $\triangle$BAC,

$\frac{AC}{AB} = \tan 30^0 \Rightarrow \frac{AC}{AB} = \frac{1}{\sqrt{3}}$

$\Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{3a}{\sqrt{3}}$

$\therefore$ AC = $\sqrt{3a}$ units

Now, volume of a cone BCD = $\frac{1}{3}\pi\,(AC)^2 \times AB$

$= \frac{1}{3}\pi(a\sqrt{3})^2 \times 3a = 3\pi a^3$

$\therefore$ Volume of water remaining in the cone = Volume of the cone BCD - Volume of a sphere

$= 3\pi a^3 - \frac{4}{3}\pi a^3 = \frac{5\pi}{3}a^3$ cu units

(ix) **(d)** 18 sq units

Explanation: {

Given, $-2\frac{2}{3} \leq x + \frac{1}{3} \leq 3\frac{1}{3}$

$\Rightarrow \frac{-8}{3} \leq x + \frac{1}{3} \leq \frac{10}{3}$

$\frac{-8}{3} \times 3 \leq \left(x + \frac{1}{3}\right)3 \leq \frac{10}{3} \times 3$ [multiplying by 3 in each term]

$\Rightarrow -8 \leq 3x + 1 \leq 10$

$\Rightarrow -8 - 1 \leq 3x + 1 - 1 \leq 10 - 1$ [subtracting 1 from each term]

$\Rightarrow -9 \leq 3x \leq 9$

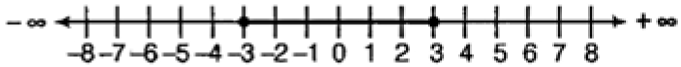
$\Rightarrow \frac{-9}{3} \leq \frac{3x}{3} \leq \frac{9}{3}$ [dividing by 3 each term]

$\Rightarrow -3 \leq x \leq 3$

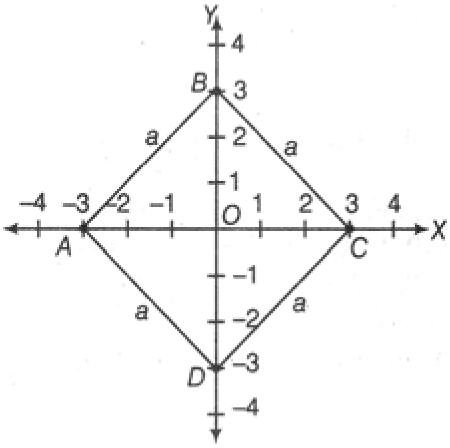
Since, $x \in R$.

$\therefore$ Range of x is [-3, 3].

Representation of range of x on the number line is given as



Now, consider the following figure



Here, AC = 6 units, which is a diagonal of square.

Let side of a square ABCD be a.

In right angled $\triangle$ABC,

$AC^2 = AB^2 + BC^2$

$\Rightarrow 6^2 = a^2 + a^2$

$\Rightarrow 36 = 2a^2$

$\Rightarrow a^2 = 18$

Now, area of a square ABCD= $(\text{Side})^2 = a^2 = 18$ sq units.

(x) **(a)** $\frac{1}{6}$

Explanation: {

In a wall clock, the minute hand cover the 60 min in on complete round.

$\therefore$ Total number of possible outcomes = 60

The minute hand cover the time from 5 to 15 min,

Number of outcomes favourable to E

= Distance from 5 to 15 min = 10

$\therefore$ Required probability = $\frac{10}{60} = \frac{1}{6}$

(xi) **(b)** $3^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explanation: {

We have, $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = 3I

$\therefore A^n = (3I)^n = 3^n I^n = 3^n I$ [$\because I^n = I$, for all natural numbers n]

$= 3^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(xii) **(a)** b^2

Explanation: {

Let P (x, y) be the moving point.

Let given two fixed points be A (a, 0) and B (-a, 0).

According to the given condition,

$PA^2 + PB^2 = 2b^2$

$\Rightarrow (x - a)^2 + (y - 0)^2 + (x + a)^2 + (y - 0)^2 = 2b^2$ [by distance formula]

$\Rightarrow x^2 - 2ax + a^2 + y^2 + x^2 + 2ax + a^2 + y^2 = 2b^2$

$\Rightarrow 2x^2 + 2y^2 + 2a^2 = 2b^2$

$\Rightarrow x^2 + y^2 + a^2 = b^2$ [dividing both sides by 2]

(xiii) **(a)** $2(\sqrt{3} + 1)r$

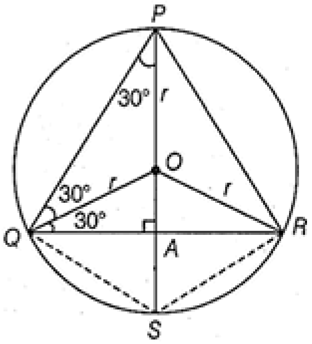
Explanation: {

As PQR is an equilateral triangle, hence PS will be perpendicular to QP and will divide it into 2 equal parts.

Since, ∠P and ∠S will be supplementary, so

$\angle S = 120^o$ and $\angle QSA = \angle RSA = 60^o$

Now, $PA = PQ \cos 30^o$ and $OA = OQ \sin 30^o = \frac{r}{2}$



$\Rightarrow AS = OA = \frac{r}{2}$ and $PA = PO + OA = r + \frac{r}{2}$

Hence, $PQ = \frac{PA}{\cos 30^o} = \frac{r + \frac{r}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}r$

In $\triangle QAS$, $AS = QS \cos 60^o \Rightarrow QS = \frac{\frac{r}{2}}{\frac{1}{2}} = r$

Since, AQ = AR, AS is common and $\angle QAS = \angle RAS = 90^o$

So, QS = RS.

∴ Perimeter of PQSP = $2(PQ + QS) = 2(\sqrt{3} + 1)r$

(xiv) **(a)** 7 : 5

Explanation: {

Mean of the observations of sets

P = $\frac{3+5+9+12+x+7+2}{7} = \frac{38+x}{7}$

Q = $\frac{8+2+1+5+7+9+3}{7} = \frac{35}{7}$ = 5

and R = $\frac{5+9+8+3+2+7+1}{7} = \frac{35}{7}$ = 5

∴ Ratio of means of sets P and Q = 7 : 5 [given]

Let P's mean = 7y and Q's mean = 5y.

⇒ 5y = 5

⇒ y = 1

∴ Q's mean = 5

Now, P's mean = 7y

⇒ $\frac{38+x}{7}$ = 7 × 1 ⇒ 38 + x = 48 ⇒ x = 11

∴ Mean of P : Mean of R = 7y : 5 = 7 × 1 : 5 = 7 : 5

(xv) **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: {

Both are correct. Reason is the correct reasoning for Assertion.

Assertion, $S_{10} = \frac{10}{2}$[2(-0.5) + (10 - 1)(-0.5)]

= 5[-1 - 4.5]

= 5(-5.5) = 27.5

2. Question 2

(i) p = ₹ 1600/month

r = 9% p.a.

m.v. = 65,592

n = ?

m.v. = $pn + \frac{p \cdot r \cdot n(n+1)}{2400}$

⇒ 65,592 = $1600n + \frac{1600 \times 9 \times n(n+1)}{2400}$

⇒ 65,592 = 1600n + 6n(n + 1)

⇒ 65,592 = 1600n + $6n^2$ + 6n

⇒ $6n^2$ + 1606n - 65,592 = 0

n = $\frac{-1606 \pm \sqrt{(1606)^2 - 4(6)(-65,592)}}{2(6)}$

n = $\frac{-1606 \pm \sqrt{4153444}}{12}$

n = $\frac{-1606 \pm 2038}{12}$

n = $\frac{-1606 + 2038}{12}$

$n = \frac{432}{12}$

n = 36 months

n = 3 years

or $n = \frac{-1606 - 2038}{12}$

or n = $\frac{-3649}{12}$

or n = -303.66 months rejected.

As 'n' is no. of months here. So can't be -ve.

(ii) Let the mean proportional between $(a^4 - b^4)^2$ and $[(a^2 - b^2)(a - b)]^{-2}$ be x.

$\Rightarrow (a^4 - b^4)^2$, x and $[(a^2 - b^2)(a - b)]^{-2}$ are in continued proportion.

$\Rightarrow (a^4 - b4)^2 : x = x : [(a^2 - b^2)(a - b)]^{-2}$

$x^2 = (a^4 - b^4)^2 \cdot [(a^2 - b^2)(a - b)]^{-2}$

$\Rightarrow x^2 = \frac{\left(a^4-b^4\right)^2}{\left[\left(a^2-b^2\right)(a-b)\right]^2}$

$\Rightarrow x = \frac{a^4-b^4}{\left(a^2-b^2\right)(a-b)}$

$\Rightarrow x = \frac{\left(a^2+b^2\right)\left(a^2-b^2\right)}{\left(a^2-b^2\right)(a-b)}$

$\Rightarrow x = \frac{a^2+b^2}{a-b}$

(iii) Given, cosec $\theta = x + \frac{1}{4x}$ ...(i)

We know that, $\cot^2 \theta = \text{cosec}^2 \theta - 1$

$\Rightarrow \cot^2 \theta = \left(x + \frac{1}{4x}\right)^2 - 1$ [from Eq. (i)]

$\Rightarrow \cot^2 \theta = x^2 + \frac{1}{16x^2} + 2x \cdot \frac{1}{4x} - 1$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

$= x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$

$= x^2 + \frac{1}{16x^2} - 2x \cdot \frac{1}{4x} = \left(x - \frac{1}{4x}\right)^2$ [$\because a^2 + b^2 - 2ab = (a - b)^2$]

$\Rightarrow \cot \theta = x - \frac{1}{4x}$ ...(ii)

or $\cot \theta = -\left(x - \frac{1}{4x}\right)$ ...(iii)

On adding Eqs. (i) and (ii), we get

cosec θ + cot θ = 2x

Now, adding Eqs. (i) and (iii), we get

cosec θ + cot $\theta = \frac{1}{2x}$

Hence, cosec θ + cot θ = 2x or $\frac{1}{2x}$.

3. Question 3

(i) Given total height of the solid = 25 cm

Height of the cone (h_2) = 4 cm

Diameter of the cylinder = 6 cm

Height of the cylinder (h_1) = 25 - 4 = 21 cm

Radius of the cone = Radius of the cylinder = (r)

$= \frac{6}{2} = 3$ cm

Slant height of cone = $\sqrt{h_2^2 + r^2}$

$= \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$

$= \sqrt{25}$

= 5 cm.

i. Volume of the solid = Volume of cylinder + Volume of cone

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \pi r^2 \left(h_1 + \frac{1}{3}h_2\right)$

$= \frac{22}{7} \times 3 \times 3 \times \left(21 + \frac{4}{3}\right)$

$= \frac{22}{7} \times 9 \times \frac{67}{3}$

$= 631.71\ cm^3 \approx 632\ cm^3$. (Approx.)

ii. Curved surface area of the solid = C.S.A of cylinder + C.S.A. of cone

$= 2\pi rh_1 + \pi rl$

$= \pi r(2h_1 + l)$

$= \frac{22}{7} \times 3(2 \times 21 + 5)$

$= \frac{22}{7} \times 3 \times 47 = 443.14\ cm^2$

Curved surface area = $443\ cm^2$ (Approx.).

(ii) Given eqn of line y = 3x - 5

Comprare with y = mx + c we get.

Slope (m) = 3 and

y-intercept (c) = -5

Now slope of the line parellel to the given line will be 3 and it passes through (0, 5).

Thus eqn of line will be

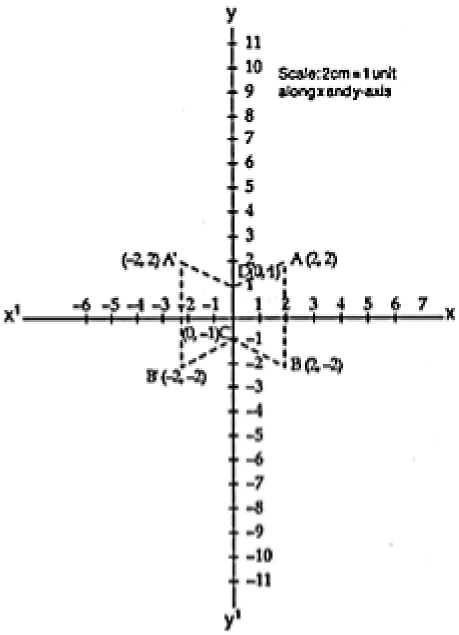
$y - y_1 = m(x - x_1)$

y - 5 = 3(x - 0)

y - 5 = 3x

Y= 3x + 5

(iii)

i. 

ii. Co-ordinates of A' → (-2, 2)

Co-ordinates of B' → (-2, - 2)

iii. Two invariant points are C (0, -1) and D (0,1).

iv. A'B'CD is an isosceles Trapezium polygon.

Section B

4. Question 4

(i) Let the List price of doll be ₹ x.

Total Amount = x + 12% of x

= x + $\frac{12}{100}x$

= $\frac{112}{100}x$

ATQ $\frac{112}{100}x$ = 3136

x = $\frac{3136\times100}{112}$

x = 2800

$\therefore$ List price of doll ₹ 2800.

Now the reduced price of the doll

= ₹(2800 - y)

amount of GST on ₹ (2800 - y)

12% of (2800 - y)

= ₹ $\frac{12}{100}$ (2800 - y)

Now the selling price of doll

= (2800 - y) + $\frac{12}{100}(2800-y)$

= (2800 - y)$\left(1+\frac{12}{100}\right)$

= (2800 - y) $\frac{112}{100}$

According to given condition, selling price of doll = list price of doll (i.e 2800)

i.e $\frac{112}{100}(2800-y)$ = 2800

2800 - y = $\frac{2800\times100}{112}$

2800 - y = 2500

2800 - 2500 = y

y = 300

Hence, amount of discount is ₹ 300.

(ii) Given equation is $x^2 + 5kx + 16 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

a = 1, b = 5k and c = 16

Now, discriminant, D = b^2 - 4ac

= $(5k)^2 - 4 \times 1 \times 16 = 25k^2 - 64$

Since, the given equation has no real roots.

$\therefore$ D < 0

$\Rightarrow 25k^2 - 64 < 0$

$\Rightarrow 25\left(k^2-\frac{64}{25}\right) < 0 \Rightarrow k^2-\frac{64}{25} < 0$

$\Rightarrow k^2 < \frac{64}{25} \Rightarrow -\frac{8}{5} < k < -\frac{8}{5}$

(iii)

Class	Frequency (f)	x	f
0-20	15	10	150
20-40	20	30	600
40-60	30	50	1500
60-80	a	70	70a
80-100	10	90	900
	Σf = 75 + a		$\sum fx$ = 3150 + 70a

Given mean = 49

$\therefore \frac{\sum fx}{\sum f} = 49$

or, $\frac{3150+70a}{75+a} = 49$

3150 + 70a = 3675 + 49a

70a - 49a = 3675 - 3150

21a = 525

a = 25

5. Question 5

(i) We know that two matrices are said to be equal if each matrix has the same number of rows and same number of columns. Corresponding elements within each matrix are equal.

Given: $\begin{bmatrix} x+y & a-b \\ a+b & 2x-3y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -1 & -5 \end{bmatrix}$

x + y = 5 ...(i),

2x - 3y = -5 ...(ii)

a - b = 3 ...(iii)

a + b = -1 ...(iv)

Solving eqn (i) and (ii)

2x + 2y = 10

2x - 3y = -5

\-   +   +

5y = 15

⇒ y = 3

Putting the value of y in eqn (i)

∴ x + 3 = 5

⇒ x = 2

Again solving eqn (iii) and (iv)

a - b = 3

a + b = -1

2a = 2

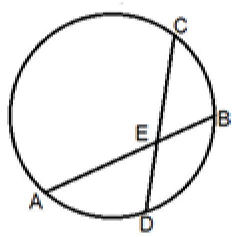
⇒ a = 1

Putting the value of a in eqn (iv)

⇒ 1 + b = -1

b = -2

⇒ x = 2, y = 3, a = 1, b = -2

(ii) 

Given, two chords AB and CD are intersect each other at point E.

AB = 9 cm

AE = 4 cm

ED = 6 cm

So, BE = AB - AE = 9 - 4 = 5

So, AE $\times$ EB = DE $\times$CE

$\Rightarrow$ 4 $\times$ 5 = 6 $\times$ CE

$\therefore$ CE = $\frac{4\times5}{6}$ = 3.34

(iii) g(x) = 0

x - 7 = 0, x = 7

By factor theorem, g(x) will be a factor of f(x)

if f(7) = 0

Now, f(7) = $(7)^3 - 6 \times (7)^2 - 19 \times 7 + 84$

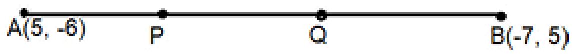
= 343 - 294 - 133 + 84

= 427 - 427 = 0

f(7) = 0, So, g(x) is a factor of f(x).

6. Question 6

(i) Let P and Q be the points of trisection of AB.



Given points be A(5, -6) and B(-7, 5)

P divides AB in the ratio 1 : 2

[By section formula, $\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}$]

the coordinate P are

$\left(\frac{1\times(-7)+2\times5}{1+2}, \frac{1\times5+2\times(-6)}{1+2}\right) = \left(\frac{-7+10}{3}, \frac{5-12}{3}\right) = \left(1, \frac{-7}{3}\right)$

$P\left(1, \frac{-7}{3}\right)$

Q divides AB in the ratio 2 : 1 then the coordinates of Q are

$\left(\frac{2\times(-7)+4\times5}{2+1}, \frac{2\times5+1\times(-6)}{2+1}\right) = \left(\frac{-14+5}{3}, \frac{10-6}{3}\right) = \left(-3, \frac{4}{3}\right)$

$Q\left(-3, \frac{4}{3}\right)$

Hence, the points of trisection of AB are $P(1, -\frac{7}{3})$ and $Q\left(-3, \frac{4}{3}\right)$.

(ii) i. LHS = $\sin^4\theta + \cos^4\theta$ = 1 - 2 $\sin^2\theta\cos^2\theta$

= $(\sin^2\theta + \cos^2\theta)^2$ - 2 $\cdot$ $\sin^2\theta\cos^2\theta$ [$\because$ $a^2 + b^2 = (a + b)^2 - 2ab$]

= 1^2 - 2 $\sin^2\theta\cos^2\theta$ = 1 - 2$\sin^2\theta\cos^2\theta$ [$\because$ $\sin^2 A + \cos^2 A = 1$]

= RHS

Hence proved.

ii. $\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta+\cot\theta}$ is true

if $\frac{1}{\operatorname{cosec}\theta-\cot\theta}+\frac{1}{\operatorname{cosec}\theta+\cot\theta}=\frac{1}{\sin\theta}+\frac{1}{\sin\theta}$ is true

i.e. if $\frac{(\operatorname{cosec}\theta+\cot\theta)+(\operatorname{cosec}\theta-\cot\theta)}{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta)}=\frac{2}{\sin\theta}$ is true

i.e. if $\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-\cot^2\theta}$ = 2 cosec θ is true

i.e. if $\frac{2\operatorname{cosec}\theta}{1}$ = 2 cosec θ is true. [∵ $\operatorname{cosec}^2\theta-\cot^2\theta=1$]

which is true.

Hence proved.

(iii)Let total work be 1 and let total work completed in days.

work dream in 1 day = $\frac{\text{Total work}}{\text{Number of days to complete work}}$

$\frac{1}{n}$

This is the work done by 150 workers

work done by 1 worker in one day = $\frac{1}{150n}$

Number of workers	work done per worker in 1 day	Total work done in 1 day
150	$\frac{1}{150n}$	$\frac{150}{150n}$
146	$\frac{1}{150n}$	$\frac{146}{150n}$
142	$\frac{1}{150n}$	$\frac{142}{150n}$

Given that, In this manner, it took 8 more days to finish the work i.e. work finished in (n + 8) days.

∴ $\frac{150}{150n}+\frac{146}{150n}+\frac{142}{150n}$ + ... + (n + 8) terms = 1

$\frac{1}{150n}$[150 + 146 + 142 + ... + (n + 8) terms] = 1

⇒ 150 + 146 + 142 + ... +(n + 8) terms = 150n

Now a = 150 d = 146 - 150

= -4

∵ diff. is equal ∴ It forms A.P.

∵ $S_n=\frac{n}{2}[2a+(n-1)d]$

∴ 150 + 146 + 142 +.. (n = 8) terms = 150n becomes.

$\frac{n+8}{2}$[2(150) + (n + 8 - 1)] = 150n

⇒ $\frac{(n+8)}{2}\times 2$ [150 - 2(n + 7)] = 150n

⇒ (n + 8)(150 - 2n - 14) = 150n

⇒ (n + 8)(136 - 2n) = 150n

⇒ $136n-2n^2+1088-16n=150n$

⇒ $2n^2-120n-1088+150n=0$

⇒ $2n^2+30n-1088=0$

⇒ $n^2+15n-544=0$

⇒ $n^2+32n-17n-544=0$

⇒ n(n + 32) - 17(n + 32) = 0

⇒ (n + 32)(n - 17) = 0

⇒ n + 32 = 0 or n = 17

n = -32 n = 17

Reject n = -32 as n should be natural no.

n = 17 work was complete in 17 + 8 = 25 days.

7. Question 7

(i) Let the two digit no. be 10x + y

product of their digits

i.e., xy = 6

$y = \frac{6}{x}$ ...(i)

According to the question

10x + y + 9 = 10y + x

9x - 9y + 9 = 0

x - y = -1 ...(ii)

Substituting the value of y from equal (i),

$\frac{x}{1} - \frac{6}{x} = -1$

$\frac{x^2-6}{x} = -1$

$x^2 - 6 = -x$

$x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

x(x + 3) - 2(x + 3) = 0

(x - 2)(x + 3) = 0

x = 2, x = - 3 (according to question rejected as digits are never negative)

put the value of x in eqn. (i)

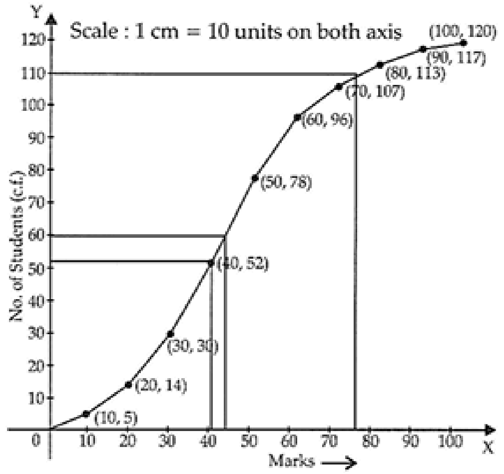
$y = \frac{6}{x} = \frac{6}{2} = 3$

Thus, x = 2 and y = 3

Hence, the required no. = 10 × 2 + 3 = 23

(ii)

C.I.	f	c.f
0 - 10	5	5
10 - 20	9	14
20 - 30	16	30
30 - 40	22	52
40 - 50	26	78
50 - 60	18	96
60 - 70	11	107
70 - 80	6	113
80 - 90	4	117
90 - 100	3	120
	N = 120	



i. Given, n = 120 which is even

∴ Median = $\frac{120}{2}$ th term

= 60th term

Median = 43

ii. The number of students who obtained more than 75% marks in test = 120 - 110 = 10.

8. Question 8

(i) Let E and F denote the events that Niharika and Shreya win the match, respectively. It is clear that, if Niharika wins the match, then Shreya losses the match and if Shreya wins the match, then Niharika losses the match. Thus, E and F are complementary events.

∴ P(E) + P(F) = 1

Since, probability of Niharika 's winning the match, i.e. P(E) = 0.62

∴ Probability of Shreya's winning the match,

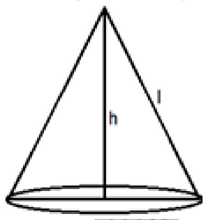
P(F) = P (Niharika losses the match)

= 1 - P(E) $[\because P(E) + P(F) = 1]$

= 1 - 0.62 = 038

(ii) Given:

h = 5m, d = 24 m, r = 12 m



$\therefore l = \sqrt{h^2 + r^2}$

$= \sqrt{5^2 + 12^2}$

l = 13 m

Convas Required = C.S.A of conical tent

$= \pi r l$

$= \frac{22}{7} \times 12 \times 13$

Convas Required = 490.28 m^2

Total cost = 490.28 × 14

= ₹6814

Hence, total cost of convas used = ₹6814

(iii) i. $\angle ABC + \angle ADC = 180^\circ$ (Opposite angles of cyclic quadrilateral)

$93^\circ + \angle ADC = 180^\circ$

$\angle ADC = 180 - 93^\circ$

$\angle ADC = 180 - 93^\circ$

= 87^o

ii. $\angle CAD = \angle ECD$ (Alternate segment theorem)

$\therefore \angle CAD = 35^\circ$

iii. In $\triangle ADC, \angle ACD + \angle CAD + \angle ADC$ = 180^o (Sum of internal angles of a triangle = 180°)

$\angle ACD + 35^\circ + 87^\circ = 180^\circ$

$\angle ACD = 180^\circ - (35^\circ + 87^\circ)$

$\angle ACD = 180^\circ - 122^\circ$

$\angle ACD = 58^\circ$

9. Question 9

(i) i. A = {x : 3 < 2x - 1 < 9, x ∈ R}

3 < 2x - 1 < 9

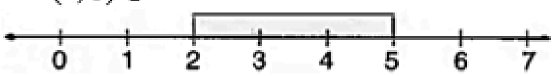
3 + 1 < 2x - 1 + 1 < 9 + 1

4 < 2x < 10

$\frac{4}{2} < \frac{2x}{2} < \frac{10}{2}$

2 < x < 5

A = (2, 5) ∈ R



B = {x : 11 ≤ 3x + 2 ≤ 23, x ∈ R}

11 ≤ 3x + 2 ≤ 23

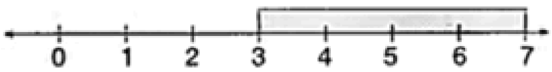
11 - 2 ≤ 3x + 2 - 2 ≤ 23 - 2

9 ≤ 3x ≤ 21

$\frac{9}{3} \leq \frac{3x}{3} \leq \frac{21}{3}$

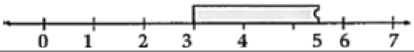
3 ≤ x ≤ 7

B = [3, 7] ∈ R



ii. A ∩ B = (2, 5) ∩ [3, 7]

= [3, 5)



(ii)

Class Interval	Required	mid value	$f_i \times x_i$
11 - 13	3	12	36
13 - 15	6	14	84
15 - 17	9	16	144
17 - 19	13	18	234
19 - 21	f	20	20f
21 - 23	5	22	110
23 - 25	4	24	96
	$\sum f_i = 40 + f$		$\sum(f_i \times x_i) = 704 + 20f$

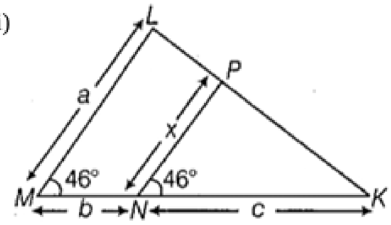
mean $(\bar{x}) = \frac{\sum(f_i \times x_i)}{\sum(f_i)}$

$18 = \frac{704+20f}{40+f}$

720 + 18f = 704 + 20f

f = 8

(iii)



Given: In the given figure.

$\angle LMN = \angle PNK = 46^o$

$\Rightarrow$ LM || PN (as corresponding angles are equal)

Now consider $\triangle LMK$ and $\triangle PNK$

$\angle LMK = \angle PNK$ (corresponding angles are equal)

$\angle LKM = \angle PKN$ (common)

$\therefore \triangle LMK \sim \triangle PNK$ (AA similarity)

$\frac{ML}{NP} = \frac{MK}{NK}$

$\frac{a}{x} = \frac{b+c}{c}$

$x = \frac{b+c}{c}$

Hence we get the result $x = \frac{ac}{b+c}$

10. Question 10

(i) Let the present age of A and B are 7x and 8x respectively.

Then, 6 years ago their ages are 7x - 6 and 8x - 6

So, $\frac{7x-6}{8x-6} = \frac{5}{6}$

$\Rightarrow 6(7x - 6) = 5(8x - 6)$

$\Rightarrow 42x - 36 = 40x - 30$

$\Rightarrow 42x - 40x = -30 + 36$

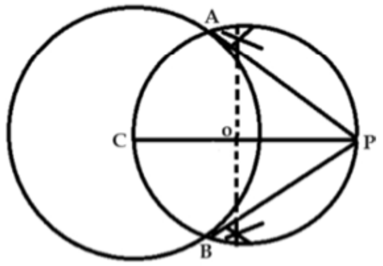
2x = 6

$\therefore x = \frac{6}{2} = 3$

Hence, the present age of A and B are 21 and 24.

(ii)
i. Draw a circle with radius 6 cm and centre C.
ii. Take a point P at 10 cm from centre and join CP.
iii. Draw perpendicular bisector of CP which cuts CP at O.
iv. Take O as centre and OC as radius draw a circle which cuts the previous circle at A and B.
v. Join PA and PB.

vi. PA and PB are required tangents.



(iii) We have, $\angle RPQ = 60^{o}$ and $\angle PQT = 30^{o}$

(iii) We have, $\angle RPQ = 60^{o}$ and $\angle PQT = 30^{o}$

and QR = 50 m

Let PT = x m and PQ = y m

In $\triangle PQR$,

$\tan 60^{o} = \frac{QR}{PQ}$

$\Rightarrow \sqrt{3} = \frac{50}{y}$

or $y = \frac{50}{\sqrt{3}}$ ...(i)

In $\triangle PQT$,

$\tan 30^{o} = \frac{PT}{PQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y}$

or $x = \frac{y}{\sqrt{3}}$ ...(ii)

From eq. (i) and (ii), we get

$x = \frac{50}{\sqrt{3}\cdot\sqrt{3}} = \frac{50}{3}$ = 16.66

= 17 m (correct to the nearest meter)