Subject Code:
YC Algebra 2

YASHWANT CLASSES Estd.1995

Subject: Algebra

Time: 2 Hours **July 2023** **[Max. Marks: 40]**

Note: i) All questions are compulsory.

ii) Use of a calculator is not allowed.

iii) The numbers on the right of the questions indicate full marks.

iv) In case of MCQs (Q.1.(A)), only the first attempt will be evaluated and will be given credit.

v) For every MCQ, the correct alternative (A), (B), (C) or (D) with sub-question number is to be written as an answer.

Q.1. (A) Choose the correct answer and write the alphabet of it in front of the sub-question number. **[4]**

(1) Sum of first five multiples of 3 is .....

(a) 45 (b) 55 (c) 15 (d) 75

(2) Find the value of determinant $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}$.

(a) 2 (b) 7 (c) –7 (d) 23

(3) Which of the following quadratic equations has roots 3 and 5?

(a) $x^2 - 15x + 8 = 0$ (b) $x^2 - 8x + 15 = 0$

(c) $x^2 + 3x + 5 = 0$ (d) $x^2 + 8x - 15 = 0$

(4) Two coins are tossed simultaneously. Write the number of sample points $n(S)$:

(a) 2 (b) 8 (c) 4 (d) 6

(B) Solve the following sub-questions. **[4]**

(1) If $15x + 17y = 21$ and $17x + 15y = 11$, then find the value of $x + y$.

(2) Given sequence is an A.P. Find the next two terms of this A.P.:

5, 12, 19, 26, ...

(3) On certain article if rate of CGST is 9%, then what is the rate of SGST and what is the rate of GST?

(4) If $n(S) = 2$ and $n(A) = 1$, then find P(A).

Q.2. (A) Complete the following activities and rewrite. (Any *two*) **[4]**

(1) Complete the following table to draw the graph of the equation $x + y = 3$:

x	3	☐	☐
y	☐	5	3
(x, y)	(3, 0)	☐	(0, 3)

(2) Complete the following activity to find the value of discriminant of the equation $x^2 + 10x - 7 = 0$.

Solution:

Comparing $x^2 + 10x - 7 = 0$ with $ax^2 + bx + c = 0$,

$a = 1, b = 10, c =$ ☐

$\therefore \quad b^2 - 4ac =$ ☐ $- 4 \times 1 \times (-7)$

$= 100 +$ ☐

$=$ ☐

(3) Complete the following table using given information:

Sr. No.	FV	Share is at	MV
1.	₹ 10	Premium of ₹ 7	☐
2.	₹ 25	☐	₹ 16
3.	₹ 300	☐	₹ 315
4.	☐	at par	₹ 5

(B) Solve the following sub-questions. (Any *four*) **[8]**

(1) Solve the following simultaneous equations:

$$x + y = 6; x - y = 4$$

(2) Solve the following quadratic equation by factorisation method:

$$x^2 + 15x + 54 = 0$$

(3) The first term $a = 8$ and common difference $d = 5$ are given. Write an A.P.

(4) Mr Rohit is a retailer. He paid GST of ₹ 6,500 at the time of purchase. He collected GST of ₹ 8,000 at the time of sale.

(a) Find his input tax and output tax.
(b) What is his input tax credit?
(c) Find his payable GST.
(d) Hence find the payable CGST and payable SGST.

(5) Find the mean from the given values:

$\Sigma x_i f_i = 1265$; N = 50

Q.3. (A) Complete the following activity and rewrite. (Any *one*) **[3]**

(1) Smita has invested ₹ 12,000 and purchased shares of FV ₹ 10 at a premium of ₹ 2. Find the number of shares she has purchased. Complete the given activity to get the answer.

Solution:

FV = ₹ 10, Premium = ₹ 2

$\therefore$ MV = FV + $\square$ = 10 + $\square$ = $\square$

$\therefore$ Number of shares = $\frac{\text{Total investment}}{\text{MV}} = \frac{12{,}000}{\square}$

= $\square$ shares

Ans. Smita has purchased $\square$ shares.

(2) If one die is rolled once, then find the probability of each of the following events:

(a) Number on the upper face is prime.

(b) Number on the upper face is even.

Solution:

'S' is the sample space.

S = {1, 2, 3, 4, 5, 6} $\therefore$ $n(S) = \square$

(a) Event A: Prime number on the upper face

A = {2, 3, 5} $\therefore$ $n(A) = \square$

$P(A) = \frac{n(A)}{n(S)}$

$\therefore$ $P(A) = \frac{3}{\square} = \square$

(b) Event B: Even number on the upper face

B = {2, 4, 6} $\therefore$ $n(B) = \square$

$P(B) = \frac{n(B)}{n(S)}$

$\therefore$ $P(B) = \square = \frac{1}{2}$

(B) Solve the following sub-questions. (Any *two*) **[6]**

(1) Two numbers differ by 3. The sum of twice the smaller number and thrice the greater number is 19. Find the numbers.

(2) Solve the given quadratic equation by using formula method:

$$5x^2 + 13x + 8 = 0$$

(3) A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets:

(a) a red balloon

(b) a blue balloon

(c) a green balloon

(4) The following table shows the number of students of class X and the time they utilized daily for their studies. Find the mean time spent by 50 students for their studies by direct method:

Time (hrs)	No. of Students
0–2	7
2–4	18
4–6	12
6–8	10
8–10	3

Q.4. Solve the following sub-questions. (Any *two*) **[8]**

(1) The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

(2) If p times the p^{th} term of an A.P. is equal to q times q^{th} term, then show that $(p + q)^{th}$ term of that A.P. is zero. $(p \neq q)$

(3) Draw a pie diagram to represent the world population given in the following table:

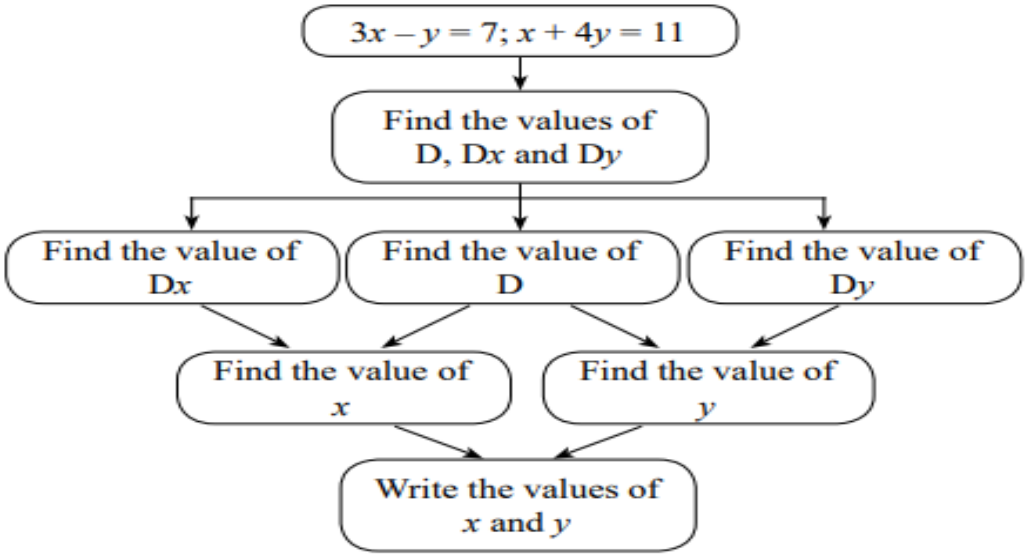
Country	Japan	England	India	China
Percentage of World Population	20	10	40	30

Q.5. Solve the following sub-question (Any *one*) **[3]**

(1) Represent the following data using histogram:

Daily Income (₹)	No. of Workers
130–135	4
135–140	7
140–145	14
145–150	16

(2) Observe the following flow chart and solve it:



Subject Code: YC Algebra 1

YASHWANT CLASSES Estd.1995

Subject: Algebra

Time: 2 Hours **July – 2023** **[Max. Marks: 40]**

Note: i) All questions are compulsory.

ii) Use of a calculator is not allowed.

iii) The numbers of the right of the questions indicates full marks.

iv) In case of MCQs (Q.1.(A)), only the first attempt will be evaluated and will be given credit.

v) For every MCQ, the correct alternative (A), (B), (C) or (D) with subquestion number is to be written as an answer.

Q.1. (A) Choose the correct answer and write the alphabet of it in front of the sub-question number. **[4]**

(1) Sum of first five multiples of 3 is ..... [1]

(a) 45 (b) 55 (c) 15 (d) 75

(2) Find the value of determinant $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}$. [1]

(a) 2 (b) 7 (c) –7 (d) 23

(3) Which of the following quadratic equations has roots 3 and 5?

(a) $x^2 - 15x + 8 = 0$ (b) $x^2 - 8x + 15 = 0$

(c) $x^2 + 3x + 5 = 0$ (d) $x^2 + 8x - 15 = 0$ [1]

(4) Two coins are tossed simultaneously. Write the number of sample points $n(S)$:

(a) 2 (b) 8 (c) 4 (d) 6 [1]

Ans. (1) – (a), (2) – (b), (3) – (b), (4) – (c)

(B) Solve the following sub-questions. **[4]**

(1) If $15x + 17y = 21$ and $17x + 15y = 11$, then find the value of $x + y$.

Solution:

$15x + 17y = 21$ ... (I)

$+ \quad 17x + 15y = 11$ ...(II)

$32x + 32y = 32$ ...[Adding equation I and II] [1/2]

Ans. $\therefore \quad x + y = 1$ (Dividing both the sides by 32) [1/2] [1]

(2) Given sequence is an A.P. Find the next two terms of this A.P.:
5, 12, 19, 26, ...

Solution:

$t_1 = 5, t_2 = 12, t_3 = 19, t_4 = 26, ...$

$d = t_2 - t_1 = 12 - 5 = 7$

$t_5 = t_4 + d = 26 + 7 = 33$ [1/2]

$t_6 = t_5 + d = 33 + 7 = 40$ [1/2] [1]

Ans. The next two terms of the given A.P. are 33 and 40.

(3) On certain article if rate of CGST is 9%, then what is the rate of SGST and what is the rate of GST?

Solution:

CGST = 9%

CGST = SGST = 9% [1/2]

Ans. GST = CGST + SGST = 9% + 9% = 18% [1/2] [1]

(4) If $n(S) = 2$ and $n(A) = 1$, then find P(A).

Solution:

$$P(A) = \frac{n(A)}{n(S)}$$ [1/2]

Ans. $\therefore P(A) = \frac{1}{2}$ [1/2] [1]

Q.2. (A) Complete the following activities and rewrite. (Any *two*) **[4]**

(1) Complete the following table to draw the graph of the equation $x + y = 3$:

Solution:

x	3	–2	0
y	0	5	3
(x, y)	(3, 0)	(–2,5)	(0, 3)

(½ mark for each blank)

(2) Complete the following activity to find the value of discriminant of the equation $x^2 + 10x - 7 = 0$.

Solution:

Comparing $x^2 + 10x - 7 = 0$ with $ax^2 + bx + c = 0$,

$a = 1, b = 10, c =$ –7 [1/2]

$\therefore \quad b^2 - 4ac =$ 10^2 $- 4 \times 1 \times (-7)$ [1/2]

$= 100 +$ 28 [1/2]

$=$ 128 [1/2][2]

(3) Complete the following table using given information:

Sr. No.	FV	Share is at	MV	
1.	₹ 10	Premium of ₹ 7	₹ 17	[1/2]
2.	₹ 25	Discount of ₹ 9	₹ 16	[1/2]
3.	₹ 300	Premium of ₹ 15	₹ 315	[1/2]
4.	₹ 5	at par	₹ 5	[1/2][2]

(B) Solve the following sub-questions. (Any *four*) **[8]**

(1) Solve the following simultaneous equations:

$x + y = 6; x - y = 4$

Solution:

$x + y = 6$ ...(I)

$+ \quad x - y = 4$ ...(II)

$2x \quad = 10$ ....{Adding equation I and II} [1/2]

$\therefore \quad x = \frac{10}{2} \quad \therefore \quad x = 5$ [1/2]

Substituting $x = 5$ in equation (I),

$5 + y = 6$

$\therefore \quad y = 6 - 5$

$\therefore \quad y = 1$ [1/2]

Ans. $x = 5$ and $y = 1$ is the solution of the given equations. [1/2] [2]

(2) Solve the following quadratic equation by factorisation method:

$x^2 + 15x + 54 = 0$

Solution:

$x^2 + 15x + 54 = 0$

$\therefore \quad x^2 + 9x + 6x + 54 = 0$ [1]

$\therefore \quad x(x + 9) + 6(x + 9) = 0$

$\therefore \quad (x + 9)(x + 6) = 0$ [1/2]

$\therefore \quad x + 9 = 0$ or $x + 6 = 0$

Ans. $\therefore \quad x = -9$ or $x = -6$ [1/2] [2]

(3) The first term $a = 8$ and common difference $d = 5$ are given. Write an A.P.

Solution:

Given that $a = 8$ and $d = 5$.

$\therefore \quad t_1 = a = 8$

$t_2 = t_1 + d = 8 + 5 = 13$ [1/2]

$t_3 = t_2 + d = 13 + 5 = 18$ [1/2]

$t_4 = t_3 + d = 18 + 5 = 23$ [1/2]

Ans. $\therefore$ The required A.P. is 8,13,18, 23,... [1/2] [2]

(4) Mr Rohit is a retailer. He paid GST of ₹ 6,500 at the time of purchase. He collected GST of ₹ 8,000 at the time of sale.

(a) Find his input tax and output tax.

(b) What is his input tax credit?

(c) Find his payable GST.

(d) Hence find the payable CGST and payable SGST.

Solution:

(a) Input tax = ₹ 6,500 (Tax paid at the time of purchase)

Output tax = ₹ 8,000 (Tax collected at the time of sale) [1/2]

(b) Input tax credit (ITC) = ₹ 6,500 [1/2]

(c) GST payable = output tax – ITC

= 8,000 – 6,500

= ₹ 1,500 [1/2]

(d) CGST = SGST = 1/2 GST

$= \frac{1}{2} \times 1{,}500$

= ₹ 750 [1/2] [2]

(5) Find the mean from the given values:
$\Sigma x_i f_i = 1265$; N = 50

Solution:

$\Sigma x_i f_i = 1265$ and N = 50 ... (Given)

$$\text{Mean} = \overline{X} = \frac{\Sigma x_i f_i}{N} \quad [1/2]$$

$$= \frac{1265}{50} \quad [1/2]$$

$$= 25.3$$

Ans. Mean = $\overline{X}$ = 25.3 [1] [2]

Q.3. (A) Complete the following activity and rewrite. (Any *one*) **[3]**

(1) Smita has invested ₹ 12,000 and purchased shares of FV ₹ 10 at a premium of ₹ 2. Find the number of shares she has purchased. Complete the given activity to get the answer.

Solution:

FV = ₹ 10, Premium = ₹ 2

∴ MV = FV + [Premium] [1/2]

= 10 + [2] = [12] [½ + ½]

$$\therefore \text{Number of shares} = \frac{\text{Total investment}}{\text{MV}}$$

$$= \frac{12{,}000}{[\ 12\ ]} \quad [1/2]$$

= [1000] shares [1/2]

Ans. Smita has purchased [1000] shares. [1/2] [3]

(2) If one die is rolled once, then find the probability of each of the following events:

(a) Number on the upper face is prime.

(b) Number on the upper face is even.

Solution:

'S' is the sample space.

S = {1, 2, 3, 4, 5, 6} ∴ $n(S)$ = [6] [1/2]

(a) Event A: Prime number on the upper face

A = {2, 3, 5} ∴ $n(A)$ = [3] [1/2]

$$P(A) = \frac{n(A)}{n(S)}$$

$\therefore \quad P(A) = \frac{3}{\boxed{6}} = \boxed{\frac{1}{2}}$ [½ + ½]

(b) Event B: Even number on the upper face

$B = \{2, 4, 6\}$

$\therefore \quad n(B) = \boxed{3}$ [1/2]

$P(B) = \frac{n(B)}{n(S)}$

$\therefore \quad P(B) = \boxed{\frac{3}{6}} = \frac{1}{2}$ [1/2] [3]

(B) Solve the following sub-questions. (Any *two*) **[6]**

(1) Two numbers differ by 3. The sum of twice the smaller number and thrice the greater number is 19. Find the numbers.

Solution:

Let the greater number be x.

Let the smaller number be y.

$\therefore$ According to the first condition,

$x - y = 3 \quad \ldots$ (I) [1/2]

According to the second condition,

$3x + 2y = 19 \quad \ldots$ (II) [1/2]

Multiplying equation (I) by 2,

$2x - 2y = 6 \quad \ldots$ (III)

$+ \quad 3x + 2y = 19 \quad \ldots$ (II) (Adding equation II and III)

$5x \quad = 25$

$\therefore \quad x = \frac{25}{5}$

$\therefore \quad x = 5$ [1/2]

Substituting $x = 5$ in equation (I),

$5 - y = 3$ [1/2]

$\therefore \quad -y = 3 - 5$

$\therefore \quad -y = -2$

$\therefore \quad y = 2$ [1/2]

Ans. The greater number is 5 and the smaller number is 2.

[1/2] [3]

(2) Solve the given quadratic equation by using formula method:

$$5x^2 + 13x + 8 = 0$$

Solution:

$5x^2 + 13x + 8 = 0$

Comparing with $ax^2 + bx + c = 0$,

$a = 5, b = 13, c = 8$ [1/2]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ [1/2]

$$= \frac{-13 \pm \sqrt{13^2 - 4 \times 5 \times 8}}{2 \times 5}$$ [1/2]

$$= \frac{-13 \pm \sqrt{169 - 160}}{10}$$

$$= \frac{-13 \pm \sqrt{9}}{10}$$

$$= \frac{-13 \pm 3}{10}$$ [1/2]

$$\therefore \quad x = \frac{-13 + 3}{10} \quad \text{or} \quad x = \frac{-13 - 3}{10}$$

$$\therefore \quad x = \frac{-10}{10} \quad \text{or} \quad x = \frac{-16}{10}$$

$$\therefore \quad x = -1 \quad \text{or} \quad x = \frac{-8}{5}$$ [1/2]

Ans. -1 and $-\frac{8}{5}$ are the roots of the given equation. [1/2] [3]

(3) A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets:

(a) a red balloon

(b) a blue balloon

(c) a green balloon

Solution:

Let the two red balloons be R_1, R_2; three blue balloons be B_1, B_2, B_3 and four green balloons be G_1, G_2, G_3, G_4. [1/2]

$\therefore S = \{R_1, R_2, B_1, B_2, B_3, G_1, G_2, G_3, G_4\}$ [1/2]

$\therefore n(S) = 9$ [1/2]

(a) Let A be the event to select a **red** balloon.

$A = \{R_1, R_2\}$

$\therefore \quad n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{2}{9}$$ [1/2]

(b) Let B be the event to select a **blue** balloon.

$$B = \{B_1, B_2, B_3\}$$

$$\therefore \quad n(B) = 3$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{9}$$

$$= \frac{1}{3} \qquad [1/2]$$

(c) Let C be the event to select a **green** balloon.

$$C = \{G_1, G_2, G_3, G_4\}$$

$$\therefore \quad n(C) = 4$$

$$P(C) = \frac{n(C)}{n(S)}$$

$$= \frac{4}{9} \qquad [1/2] \; [3]$$

(4) The following table shows the number of students of class X and the time they utilized daily for their studies. Find the mean time spent by 50 students for their studies by direct method:

Time (hrs)	No. of Students
0–2	7
2–4	18
4–6	12
6–8	10
8–10	3

Solution:

Time (hrs)	Class mark (x_i)	Frequency (f_i)	$x_i f_i$
0–2	1	7	7
2–4	3	18	54
4–6	5	12	60
6–8	7	10	70
8–10	9	3	27
Total		$\Sigma f_i = 50$	$\Sigma x_i f_i = 218$

[each correct column 1/2 mark]

$$\text{Mean} = \overline{X} = \frac{\Sigma x_i f_i}{\Sigma f_i} \qquad [1/2]$$

$$= \frac{218}{50}$$

$$= 4.36 \qquad [1/2][3]$$

Ans. The mean time spent by students is 4.36 hrs.

Q.4. Solve the following sub-questions. (Any *two*) **[8]**

(1) The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

Solution:

Let α and β are the roots of the equation.

$\therefore \quad \alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$ [½ + ½]

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ [1]

$\therefore \quad 35 = (5)^3 - 3\alpha\beta \times 5$ [1/2]

$\therefore \quad 35 = 125 - 15\alpha\beta$

$\therefore \quad 15\alpha\beta = 125 - 35$

$\therefore \quad \alpha\beta = \frac{90}{15}$

$\therefore \quad \alpha\beta = 6$ [1/2]

The required quadratic equation is

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$ [1/2]

Ans. $\therefore \quad x^2 - 5x + 6 = 0$ [1/2] [4]

(2) If p times the p^{th} term of an A.P. is equal to q times q^{th} term, then show that $(p+q)^{th}$ term of that A.P. is zero. $(p \neq q)$

Solution:

Let a be the first term and d be the common difference of the given A.P.

$p \times t_p = q \times t_q$ ... (given) [1/2]

$t_n = a + (n-1)d$

$\therefore \quad t_p = a + (p-1)d$ and $t_q = a + (q-1)d$

$\therefore \quad p[a + (p-1)d] = q[a + (q-1)d]$ [1/2]

$\therefore \quad p[a + (p-1)d] - q[a + (q-1)d] = 0$

$\therefore \quad ap + p(p-1)d - aq - q(q-1)d = 0$ [1/2]

$\therefore \quad ap - aq + p(p-1)d - q(q-1)d = 0$

$\therefore \quad a(p-q) + (p^2 - p)d - (q^2 - q)d = 0$

$\therefore \quad a(p-q) + d[(p^2 - p) - (q^2 - q)] = 0$

$\therefore \quad a(p-q) + d(p^2 - p - q^2 + q) = 0$

$\therefore \quad a(p-q) + d[(p^2 - q^2) - (p-q)] = 0$

$\therefore \quad a(p-q) + d[(p+q)(p-q) - (p-q)] = 0$ [1/2]

$\therefore \quad a(p-q) + d[(p-q)(p+q-1)] = 0$

$\therefore \quad (p-q)[a + d(p+q-1)] = 0$ [1/2]

But $(p-q) \neq 0$

$\therefore \quad a + d(p+q-1) = 0$ ...(I) [1/2]

But $t_{(p+q)} = a + d(p+q-1)$ ...(II) [1/2]

$\therefore$ From (I) and (II),

$t_{(p+q)} = 0$ [1/2] [4]

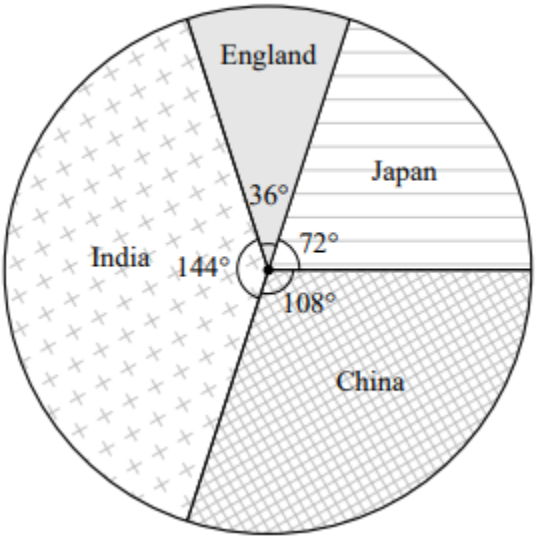
(3) Draw a pie diagram to represent the world population given in the following table:

Country	Japan	England	India	China
Percentage of World Population	20	10	40	30

Solution:

Country	% of population	Measure of central angle
Japan	**20**	$360° \times \frac{20}{100} = 36° \times 2 = 72°$
England	**10**	$360° \times \frac{10}{100} = 36° \times 1 = 36°$
India	**40**	$360° \times \frac{40}{100} = 36° \times 4 = 144°$
China	**30**	$360° \times \frac{30}{100} = 36° \times 3 = 108°$
Total	**100**	**360°**

[Each correct angle 1/2 mark]

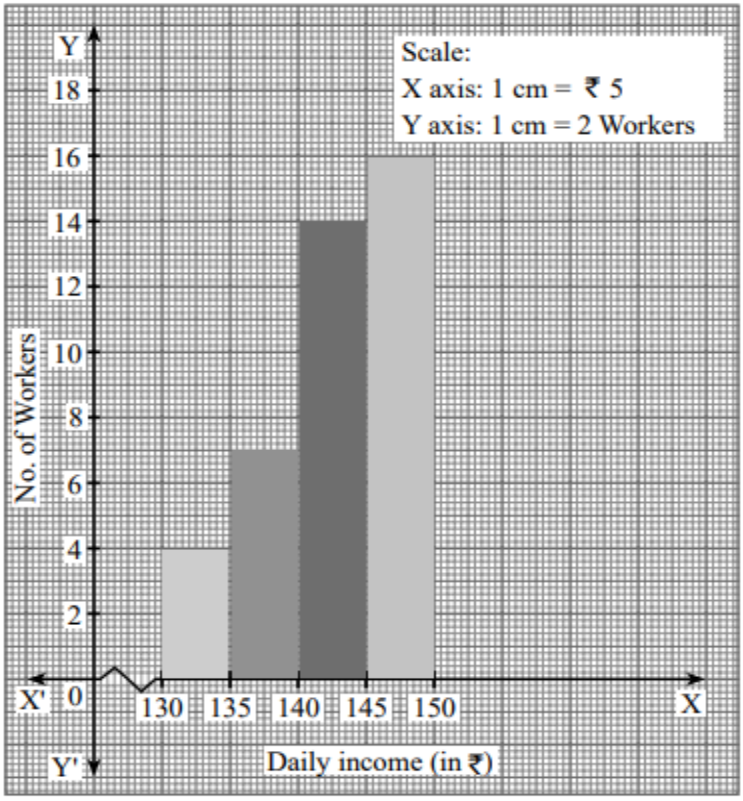


Q.5. Solve the following sub-question. (Any *one*) **[3]**

(1) Represent the following data using histogram:

Daily Income (₹)	No. of Workers
130–135	4
135–140	7
140–145	14
145–150	16

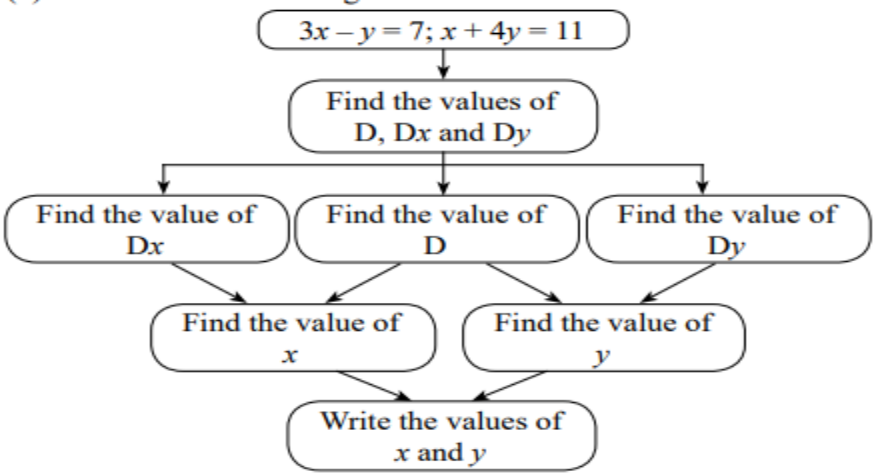
Solution:



[for each correct bar 1/2 mark] [2]

[for correct scale] [1] [3]

(2) Observe the following flow chart and solve it:



Solution:

$$3x - y = 7; x + 4y = 11$$

$$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = 3 \times 4 - (-1) \times 1$$
$$= 12 + 1$$
$$= 13 \qquad [1/2]$$

$$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = 7 \times 4 - (-1) \times 11$$
$$= 28 + 11$$
$$= 39 \qquad [1/2]$$

$$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = 3 \times 11 - 7 \times 1$$
$$= 33 - 7$$
$$= 26 \qquad [1/2]$$

$$x = \frac{D_x}{D} = \frac{39}{13} = 3 \qquad [1/2]$$

$$y = \frac{D_y}{D} = \frac{26}{13} = 2 \qquad [1/2]$$

Ans. $x = 3, y = 2$ [1/2] [3]

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