[redacted]

YASHWANT CLASSES Estd.1995

Subject: [redacted]

Time: 2 Hours **MARCH – 2022** **[Max. Marks: 40]**

Note: i) All questions are compulsory.

ii) Use of a calculator is not allowed.

iii) The numbers of the right of the questions indicates full marks.

iv) In case of MCQs (Q.1.(A)), only the first attempt will be evaluated and will be given credit.

v) For every MCQ, the correct alternative (A), (B), (C) or (D) with sub question number is to be written as an answer.

Q.1. (A) For each of the following sub-questions four alternative answers are given. Choose the correct alternative and write its alphabet: [4]

(1) If $\Delta ABC \sim \Delta DEF$ and $\angle A = 48°$, then $\angle D$ = ____________.

(a) $48°$ (b) $83°$ (c) $49°$ (d) $132°$

(2) AP is a tangent at A drawn to the circle with centre O from an external point P. OP = 12 cm and $\angle OPA = 30°$, then the radius of the circle is ____________.

(a) 12 cm (b) $6\sqrt{3}$ cm (c) 6 cm (d) $12\sqrt{3}$ cm

(3) Seg AB is parallel to X-axis and co-ordinates of the point A are (1, 3), then the co-ordinates of the point B can be ____________.

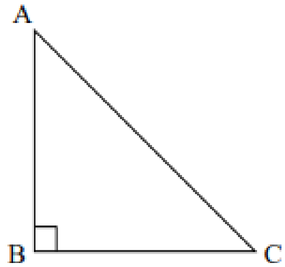
(a) (–3, 1) (b) (5, 1) (c) (3, 0) (d) (–5, 3)

(4) The value of $2\tan 45° - 2\sin 30°$ is ____________.

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Q.1. (B) Solve the following sub-questions: [4]

(1)

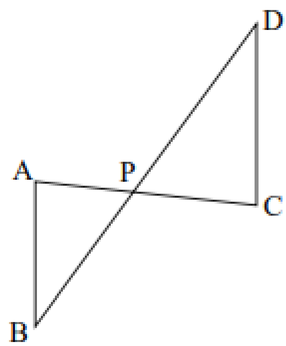


In ΔABC, $\angle ABC = 90°$, $\angle BAC = \angle BCA = 45°$. If $AC = 9\sqrt{2}$, then find the value of AB.

(2) Chord AB and chord CD of a circle with centre O are congruent. If $m(\text{arc } AB) = 120°$, then find $m(\text{arc } CD)$.

Q.2. (A) Complete the following activities and rewrite them (any *two*): [4]

(1) In the alongside figure, seg AC and seg BD intersect each other in point P. If $\frac{AP}{CP} = \frac{BP}{DP}$, then complete the following activity to prove $\Delta ABP \sim \Delta CDP$.



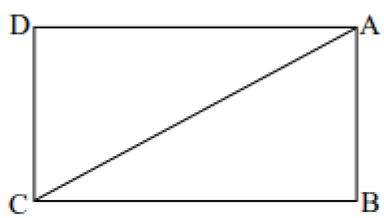
Activity: In ΔAPB and ΔCDP

$\frac{AP}{CP} = \frac{BP}{DP}$ ...$\square$

$\therefore$ $\angle APB \cong \square$ ...Vertically opposite angles

$\therefore$ $\square \sim \Delta CDP$ ...$\square$ test of similarity.

(2) In the alongside figure, $\square$ABCD is a rectangle. If AB = 5, AC = 13, then complete the following activity to find BC.



Activity:

ΔABC is $\square$ triangle.

$\therefore$ By Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$\therefore \quad 25 + BC^2 = \square$

$\therefore \quad BC^2 = \square$

$\therefore \quad BC = \square$

(3) Complete the following activity to prove:

$\cot\theta + \tan\theta = \operatorname{cosec}\theta \times \sec\theta$

Activity:

L.H.S. $= \cot\theta + \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\square}{\cos\theta}$

$= \dfrac{\square + \sin^2\theta}{\sin\theta \times \cos\theta}$

$= \dfrac{1}{\sin\theta \times \cos\theta}$ .... $\because \square$

$= \dfrac{1}{\sin\theta} \times \dfrac{1}{\cos\theta}$

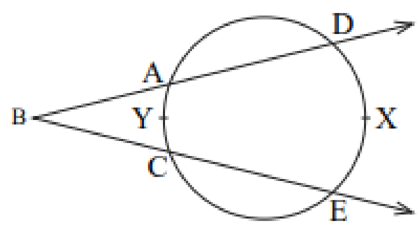
$= \square \times \sec\theta$

$\therefore$ L.H.S. = R.H.S.

Q.2. (B) Solve the following sub-questions (any *four*): [8]

(1) If $\Delta ABC \sim \Delta PQR$, $AB:PQ = 4:5$ and $A(\Delta PQR) = 125\ \text{cm}^2$, then find $A(\Delta ABC)$.

(2) In the following figure, $m(\text{arc } DXE) = 105°$, $m(\text{arc } AYC) = 47°$, then find the measure of $\angle DBE$.



(3) Draw a circle of radius 3.2 cm and centre 'O'. Take any point P on it. Draw a tangent to the circle through point P using the centre of the circle.

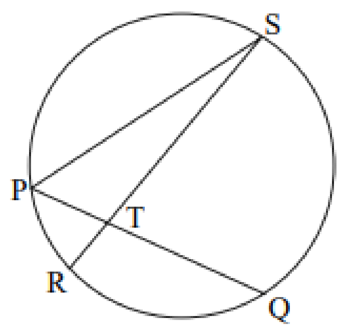
(4) If $\sin \theta = \frac{11}{61}$, then find the value of $\cos \theta$ using trigonometric identity.

(5) In ΔABC, AB = 9 cm, BC = 40 cm, AC = 41 cm. State whether ΔABC is a right-angled triangle or not. Write reason.

Q.3. (A) Complete the following activity and rewrite it (any *one*): **[3]**

(1) In the alongside figure, chord PQ and chord RS intersect each other at point T. If $\angle STQ = 58°$ and $\angle PSR = 24°$, then complete the following activity to verify:

$\angle STQ = \frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})]$



Activity:

In ΔPTS,

$\angle SPQ = \angle STQ - \boxed{}$ .....Exterior angle theorem

$\therefore \quad \angle SPQ = 34°$

$\therefore \quad m(\text{arc QS}) = 2 \times \boxed{}° = 68°$ .... $\boxed{}$

Similarly, $m(\text{arc PR}) = 2\angle PSR = \boxed{}°$

$\therefore \quad \frac{1}{2}[m(\text{arc QS}) + m(\text{arc PR})] = \frac{1}{2} \times \boxed{}° = 58°$ .... (I)

But $\angle STQ = 58°$ .... (II), given

$\therefore \quad \frac{1}{2}[m(\text{arc PR}) + m(\text{arc QS})] = \boxed{\angle....}$ .... From (I) and (II)

(2) Complete the following activity to find the co-ordinates of point P which divides seg AB in the ratio 3:1 where A(4, –3) and B(8, 5).

Activity:

By section formula,

$$x = \frac{mx_2 + nx_1}{\boxed{}}, \qquad y = \frac{\boxed{}}{m + n}$$

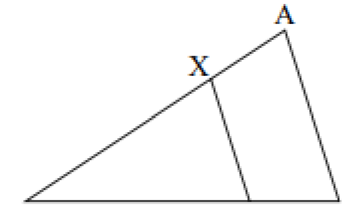
$$\therefore \quad x = \frac{3 \times 8 + 1 \times 4}{3 + 1}, \qquad y = \frac{3 \times 5 + 1 \times (-3)}{3 + 1}$$

$$= \frac{\square + 4}{4}, \qquad = \frac{\square - 3}{4}$$

$$\therefore \quad x = \square, \qquad \therefore \quad y = \square$$

Q.3. (B) Solve the following sub-questions (any *two*): [6]

(1) In ΔABC, seg $XY \parallel$ side AC. If $2AX = 3BX$ and $XY = 9$, then find the value of AC.



(2) Prove that "Opposite angles of cyclic quadrilateral are supplementary."

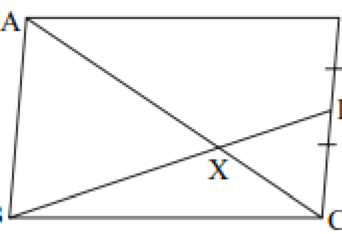
(3) $\Delta ABC \sim \Delta PQR$. In ΔABC, $AB = 5.4$ cm, $BC = 4.2$ cm, $AC = 6.0$ cm, $AB:PQ = 3:2$, then construct ΔABC and ΔPQR.

(4) Show that:

$$\frac{\tan A}{(1 + \tan^2 A)^2} + \frac{\cot A}{(1 + \cot^2 A)^2} = \sin A \times \cos A$$

Q.4. Solve the following sub-questions (any *two*): [8]

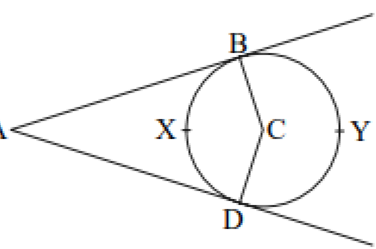
(1)



□ABCD is a parallelogram. Point P is the midpoint of side CD. Seg BP intersects diagonal AC at point X, then prove that:

$$3AX = 2AC$$

(2) In the alongside figure, seg AB and seg AD are tangent segments drawn to a circle with centre C from exterior point A, then prove that:



$$\angle A = \frac{1}{2}[m(\text{arc } BYD) - m(\text{arc } BXD)]$$

(3) Find the co-ordinates of centroid of a triangle if points D(–7, 6), E(8, 5) and F(2, –2) are the midpoints of the sides of that triangle.

Q.5. Solve the following sub-question (any *one*): **[3]**

(1) a and b are natural numbers and $a > b$. If $(a^2 + b^2)$, $(a^2 - b^2)$ and $2ab$ are the sides of a triangle, then prove that the triangle is right angled.

Find out two Pythagorean triplets by taking suitable values of a and b.

(2) Construct two concentric circles with centre O and radii 3 cm and 5 cm. Construct a tangent to the smaller circle from any point A on the larger circle. Measure and write the length of the tangent segment. Calculate the length of the tangent segment using Pythagoras theorem.

***

Time: 2 Hours **MARCH – 2023** **[Max. Marks: 40]**

Q.1. (A) For each of the following sub-questions four alternative answers are given. Choose the correct alternative and write its alphabet: **[4]**

(1) If $\Delta ABC \sim \Delta DEF$ and $\angle A = 48°$, then $\angle D$ = ____________.

(a) 48° (b) 83° (c) 49° (d) 132°

(2) AP is a tangent at A drawn to the circle with centre O from an external point P. OP = 12 cm and $\angle OPA = 30°$, then the radius of the circle is ____________.

(a) 12 cm (b) $6\sqrt{3}$ cm (c) 6 cm (d) $12\sqrt{3}$ cm

(3) Seg AB is parallel to X-axis and co-ordinates of the point A are (1, 3), then the co-ordinates of the point B can be ____________.

(a) (–3, 1) (b) (5, 1) (c) (3, 0) (d) (–5, 3)

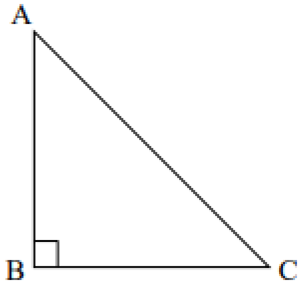
(4) The value of $2\tan 45° - 2\sin 30°$ is ____________.

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Ans. (1) - (a), (2) - (c), (3) - (d), (4) - (b)

Q.1. (B) Solve the following questions. **[4]**

(1)



In ΔABC, $\angle ABC = 90°$, $\angle BAC = \angle BCA = 45°$. If $AC = 9\sqrt{2}$, then find the value of AB.

Solution:

In ΔABC, $\angle ABC = 90°$
$\angle BAC = \angle BCA = 45°$ } ....(Given)

Let $AB = BC = x$

By Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$\therefore \quad x^2 + x^2 = (9\sqrt{2})^2$

$\therefore \quad 2x^2 = 81 \times 2$

$\therefore \quad x^2 = 81$

$\therefore \quad x = 9$

$\therefore \quad AB = 9$

(2) Chord AB and chord CD of a circle with centre O are congruent. If *m*(arc AB) = 120°, then find *m*(arc CD).

Solution:

Chord CD $\cong$ Chord AB ...(Given)

$\therefore$ *m*(arc CD) = *m*(arc AB) ...(Corresponding arcs of congruent chords)

= 120° ...[As *m*(arc AB) = 120°, given]

$\therefore$ *m*(arc CD) = 120°

(3) Find the Y co-ordinate of the centroid of a triangle whose vertices are (4, –3), (7, 5) and (–2, 1).

Solution:

Let $(4, -3) \equiv (x_1, y_1)$

$(7, 5) \equiv (x_2, y_2)$

$(-2, 1) \equiv (x_3, y_3)$

By the centroid formula,

$$y = \frac{y_1 + y_2 + y_3}{3}$$

$$= \frac{-3 + 5 + 1}{3}$$

$$= \frac{3}{3}$$

$$= 1$$

Ans. The Y co-ordinate of the centroid of the triangle is 1.

(4) If sin θ = cos θ, then what will be the measure of angle θ?

Solution:

$\sin\theta = \cos\theta$ (Given)

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

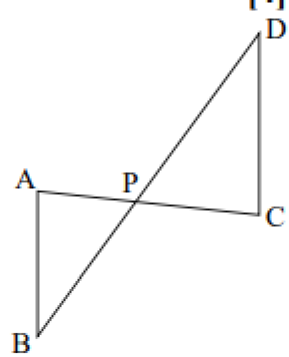
$\therefore \theta = 45°$

Q.2. (A) Complete the following activities and rewrite them (any *two*): [4]

(1) In the alongside figure, seg AC and seg BD intersect each other in point P.

If $\frac{AP}{CP} = \frac{BP}{DP}$, then complete the following activity to prove

$\Delta ABP \sim \Delta CDP$.



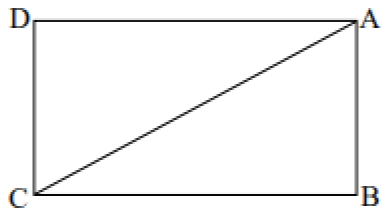
Activity: In ΔAPB and ΔCDP,

$\frac{AP}{CP} = \frac{BP}{DP}$ ... Given

$\therefore \angle APB \cong$ ∠CPD ...Vertically opposite angles

$\therefore$ ΔABP ~ ΔCDP ... SAS test of similarity

(2) In the alongside figure, □ABCD is a rectangle. If AB = 5, AC = 13, then complete the following activity to find BC.



Activity:

ΔABC is [right-angled] triangle.

$\therefore$ By Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$\therefore$ $25 + BC^2 = \boxed{169}$

$\therefore$ $BC^2 = \boxed{144}$

$\therefore$ $BC = \boxed{12}$

(3) Complete the following activity to prove:

$\cot\theta + \tan\theta = \operatorname{cosec}\theta \times \sec\theta$

Activity:

L.H.S. $= \cot\theta + \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\boxed{\sin\theta}}{\cos\theta}$$

$$= \frac{\boxed{\cos^2\theta} + \sin^2\theta}{\sin\theta \times \cos\theta}$$

$$= \frac{1}{\sin\theta \times \cos\theta} \quad \text{..........} \quad \boxed{\because \ \sin^2\theta + \cos^2\theta = 1}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

$$= \boxed{\operatorname{cosec}\theta} \times \sec\theta$$

$\therefore$ L.H.S. = R.H.S.

Q.2. (B) Solve the following sub-questions (any *four*): [8]

(1) If $\Delta ABC \sim \Delta PQR$, AB:PQ = 4:5 and $A(\Delta PQR) = 125\ cm^2$, then find $A(\Delta ABC)$.

Solution:

$\Delta ABC \sim \Delta PQR$ ...(Given)

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ... (Theorem of area of two similar triangles)

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$

$\therefore \frac{A(\Delta ABC)}{125} = \left(\frac{4}{5}\right)^2$

$\therefore \frac{A(\Delta ABC)}{125} = \frac{16}{25}$

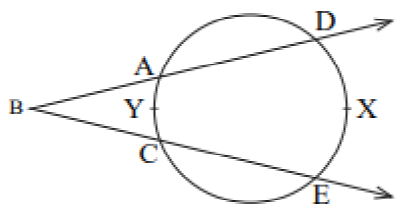
$\therefore A(\Delta ABC) = \frac{16 \times 125}{25}$

$= 16 \times 5$

$= 80$

$\therefore A(\Delta ABC) = 80$ sq.cm

(2) In the following figure, *m*(arc DXE) = 105°, *m*(arc AYC) = 47°, then find the measure of ∠DBE.



Solution:

$\angle DBE = \frac{1}{2}[m(\text{arc DXE}) - m(\text{arc AYC})]$

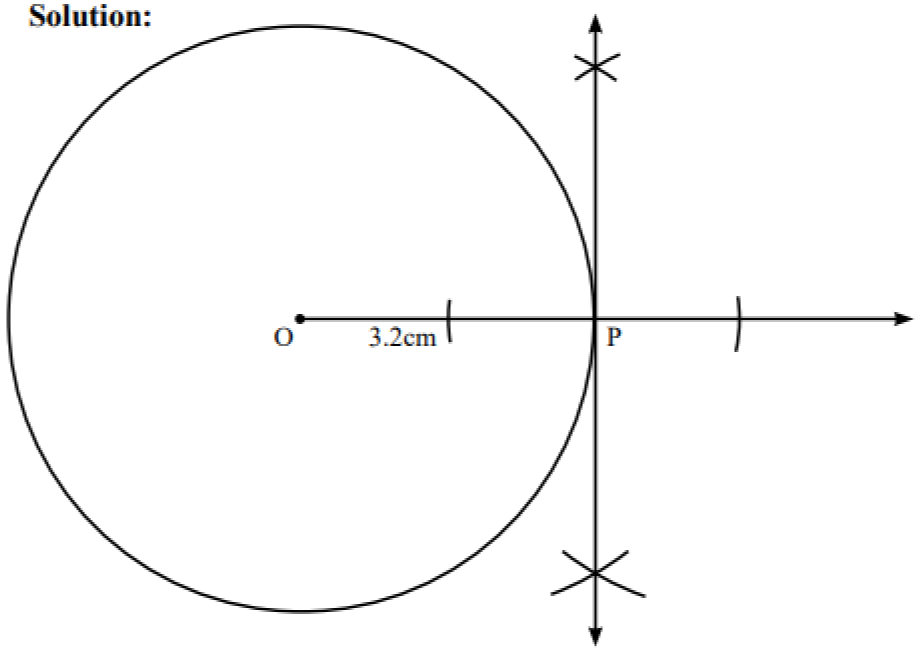
$= \frac{1}{2}[105° - 47°]$

$= \frac{1}{2} \times 58°$

$\therefore \angle DBE = 29°$

(3) Draw a circle of radius 3.2 cm and centre 'O'. Take any point P on it. Draw a tangent to the circle through point P using the centre of the circle.

Solution:



(4) If $\sin\theta = \frac{11}{61}$, then find the value of $\cos\theta$ using trigonometric identity.

Solution:

$\sin^2\theta + \cos^2\theta = 1$ ...(Trigonometric identity)

$\therefore \left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$

$\therefore \frac{121}{3721} + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \frac{121}{3721}$

$= \frac{3721 - 121}{3721}$

$= \frac{3600}{3721}$

$\therefore \cos\theta = \frac{60}{61}$ ...(Taking square root on both sides)

(5) In ΔABC, AB = 9 cm, BC = 40 cm, AC = 41 cm. State whether ΔABC is a right-angled triangle or not. Write reason.

Solution:

In ΔABC,

$AB^2 + BC^2 = 9^2 + (40)^2$

$= 81 + 1600$

$= 1681$ ...(i)

$AC^2 = (41)^2 = 1681$ ...(ii)

$\therefore$ $AB^2 + BC^2 = AC^2$ ...[From (i) and (ii)]

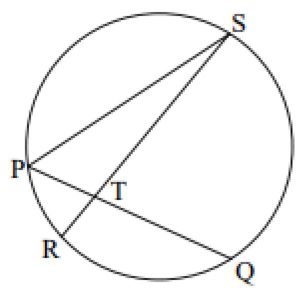
$\therefore$ ΔABC is a right-angled tringle.

(By converse of Pythagoras theorem)

Q.3. (A) Complete the following activity and rewrite it. (Any *one*): **[3]**

(1) In the alongside figure, chord PQ and chord RS intersect each other at point T. If $\angle STQ = 58°$ and $\angle PSR = 24°$, then complete the following activity to verify:

$\angle STQ = \frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})]$



Activity:

In ΔPTS,

$\angle SPQ = \angle STQ - \boxed{\angle PST}$ (Exterior angle theorem)

$\therefore$ $\angle SPQ = 34°$

$\therefore$ $m(\text{arc QS}) = 2 \times \boxed{34}° = 68°$ ...(Inscribed angle theorem)

Similarly, $m(\text{arc PR}) = 2\angle PSR = \boxed{48}°$

$\therefore$ $\frac{1}{2}[m(\text{arc QS}) + m(\text{arc PR})] = \frac{1}{2} \times \boxed{68 + 48}° = 58°$ ...(I)

But $\angle STQ = 58°$ ...(II), given

$\therefore$ $\frac{1}{2}[m(\text{arc PR}) + m(\text{arc QS})] = \boxed{\angle STQ}$ ...From (I) and (II)

(2) Complete the following activity to find the co-ordinates of point P which divides seg AB in the ratio 3:1 where A(4, –3) and B(8, 5).

Activity:

By section formula,

$$x = \frac{mx_2 + nx_1}{\boxed{m+n}}, \qquad y = \frac{\boxed{my_2 + ny_1}}{m+n}$$

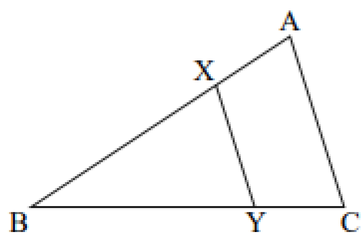
$$\therefore \quad x = \frac{3 \times 8 + 1 \times 4}{3+1}, \qquad y = \frac{3 \times 5 + 1 \times (-3)}{3+1}$$

$$= \frac{\boxed{24} + 4}{4}, \qquad = \frac{\boxed{15} - 3}{4}$$

$$\therefore \quad x = \boxed{7} \qquad \therefore\ y = \boxed{3}$$

Q.3. (B) Solve the following sub-questions (any *two*): [6]

(1) In ΔABC, seg $XY \parallel$ side AC. If $2AX = 3BX$ and $XY = 9$, then find the value of AC.



Solution:

In ΔABC,

$$\left.\begin{aligned} \text{seg } XY &\parallel \text{side } AC \\ 2AX &= 3BX \end{aligned}\right\} \quad \text{...(Given)}$$

$$\therefore \quad \frac{AX}{BX} = \frac{3}{2}$$

$$\therefore \quad \frac{AX + BX}{BX} = \frac{3+2}{2} \qquad \text{...(By Componendo)}$$

$$\therefore \quad \frac{AB}{BX} = \frac{5}{2} \qquad \text{...(i) } (\because A\text{–}X\text{–}B)$$

In ΔBCA and ΔBYX,

$$\angle BCA \cong \angle BYX \qquad \text{...(Corresponding angles)}$$

$$\angle CBA \cong \angle YBX \qquad \text{...(Common angle)}$$

$$\therefore \quad \Delta BCA \cong \Delta BYX \qquad \text{...(AA test of similarity)}$$

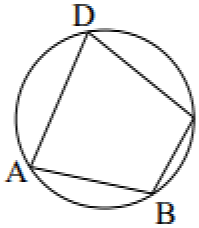
$$\therefore \quad \frac{AB}{BX} = \frac{AC}{XY} \qquad \text{...(C.S.S.T)}$$

$$\therefore \quad \frac{5}{2} = \frac{AC}{9}$$

$$\therefore \quad AC = \frac{5 \times 9}{2}$$

$$\therefore \quad AC = 22.5$$

(2) Prove that "Opposite angles of cyclic quadrilateral are supplementary."



Solution:

Given: □ABCD is cyclic.

To prove:: $\angle B + \angle D = 180°$

$\angle A + \angle C = 180°$

Proof:

Arc ABC is intercepted by the inscribed angle ADC.

$$\therefore \quad \angle ADC = \frac{1}{2} m(\text{arc } ABC) \qquad \text{...(i)}$$

Similarly, arc ADC is intercepted by the insctribed angle ABC.

$$\therefore \quad \angle ABC = \frac{1}{2} m(\text{arc } ADC) \qquad \text{...(ii)}$$

$$\therefore \quad m\angle ADC + m\angle ABC = \frac{1}{2} m(\text{arc } ABC) + \frac{1}{2} m(\text{arc } ADC)$$

[From (i) and (ii)]

$$= \frac{1}{2} [m(\text{arc } ABC) + m(\text{arc } ADC)]$$

$$= \frac{1}{2} \times 360° \qquad \text{...(Arcs ABC and ADC constitute a complete circle)}$$

$$\therefore \quad \angle ADC + \angle ABC = 180°$$

Similarly, we can prove that

$$\angle A + \angle C = 180°.$$

Similarly, we can prove that

$$\angle A + \angle C = 180°.$$

(3) $\Delta ABC \sim \Delta PQR$. In ΔABC, AB = 5.4 cm, BC = 4.2 cm, AC = 6.0 cm, AB:PQ = 3:2, then construct ΔABC and ΔPQR.

Solution:

$\Delta ABC \sim \Delta PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$ ...(C.S.S.T)

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR} = \dfrac{3}{2}$ ...(AB:PQ = 3:2, given)

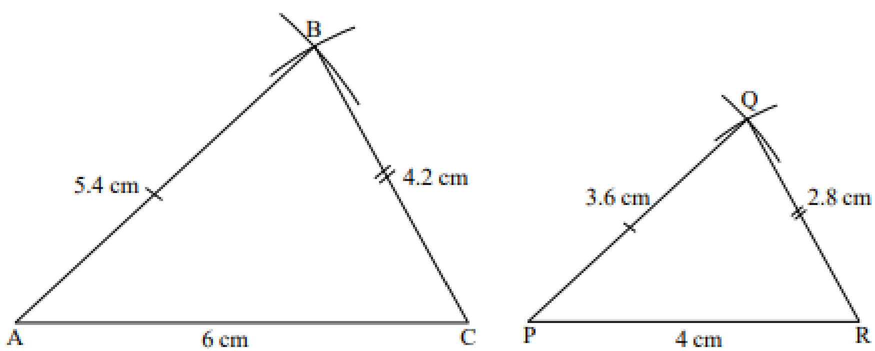
$\therefore \dfrac{5.4}{PQ} = \dfrac{4.2}{QR} = \dfrac{6}{PR} = \dfrac{3}{2}$

$\therefore \dfrac{5.4}{PQ} = \dfrac{3}{2}$

$\therefore PQ = \dfrac{5.4 \times 2}{3} = 3.6$ cm

$\dfrac{4.2}{QR} = \dfrac{3}{2}$

$\therefore QR = \dfrac{4.2 \times 2}{3} = 2.8$ cm



(4) Show that:

$$\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2} = \sin A \times \cos A$$

Solution:

$$\text{LHS} = \frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2}$$

$$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\text{cosec}^2 A)^2}$$ ...($\because$ $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$)

$$= \frac{\sin A}{\cos A} \times (\cos^2 A)^2 + \frac{\cos A}{\sin A} \times (\sin^2 A)^2$$

$$= \sin A \times \cos^3 A + \cos A \times \sin^3 A$$

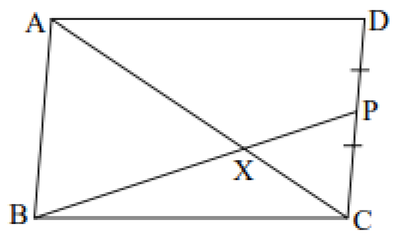
$$= \sin A.\cos A \ (\cos^2 A + \sin^2 A)$$

$$= \sin A.\cos A \qquad ...(\because \ \sin^2 \theta.\ \cos^2 \theta = 1)$$

$\therefore$ LHS = RHS

Q.4. Solve the following sub-questions (any *two*) **[8]**

(1)



□ABCD is a parallelogram. Point P is the midpoint of side CD. Seg BP intersects diagonal AC at point X, then prove that: 3AX = 2AC

Solution:

Proof: □ABCD is parallelogram, and point P is the midpoint of side DC. ...(Given)

$\therefore$ AB = CD = 2CP ...(i)

Now, in ΔAXB and ΔCXP,

$\angle AXB \cong \angle CXP$ ...(Vertically opposite angles)

$\angle BAX \cong \angle PCX$ ...(Alternate angles)

$\therefore \ \Delta AXB \sim \Delta CXP$ ...(By AA test)

$\therefore \ \frac{AX}{CX} = \frac{AB}{CP}$ ...(C.S.S.T)

$\therefore \ \frac{AX}{CX} = \frac{2CP}{CP}$ ...[From (i)]

$\therefore \ \frac{AX}{CX} = \frac{2}{1}$

$\therefore \ CX = \frac{AX}{2}$

Now, AC = AX + CX ...(A–X–C)

$\therefore \ AC = AX + \frac{AX}{2}$

$\therefore \quad AC = \frac{2AX + AX}{2}$

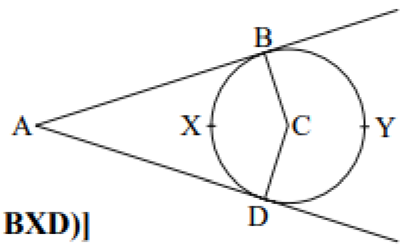
$\therefore \quad 2AC = 3AX$

$\therefore \quad 3AX = 2AC$

Hence proved.

(2) In the alongside figure, seg AB and seg AD are tangent segments drawn to a circle with centre C from exterior point A, then prove that:

$\angle A = \frac{1}{2}$ [m(arc BYD) – m(arc BXD)]



Solution:

seg CB ⊥ seg AB
seg CD ⊥ seg AD } (Tangent perpendicular to radius)

$\therefore \quad \angle ABC = \angle ADC = 90°$ ...(i)

Now, in □ABCD,

$\angle A + \angle C + \angle B + \angle D = 360°$ ...(Sum of angles of quadrilateral)

$\therefore \quad \angle A + \angle C + 90° + 90 = 360°$ ...[From (i)]

$\therefore \quad \angle A + \angle C = 360° - 180°$

$\therefore \quad \angle A + \angle C = 180°$

$\therefore \quad \angle A = 180° - \angle C$

But $\angle C = m(\text{arc BXD})$ ....(Definition of measure of arc)

$\therefore \quad \angle A = 180° - m(\text{arc BXD})$ ...(ii)

Now,

$m(\text{arc BXD}) + m(\text{arc BYD}) = 360°$ ...(Measure of complete circle)

$\therefore \quad \frac{1}{2} m(\text{arc BXD}) + \frac{1}{2} m(\text{arc BYD}) = 180°$ ...(iii) ...[Multiplying (ii) by $\frac{1}{2}$]

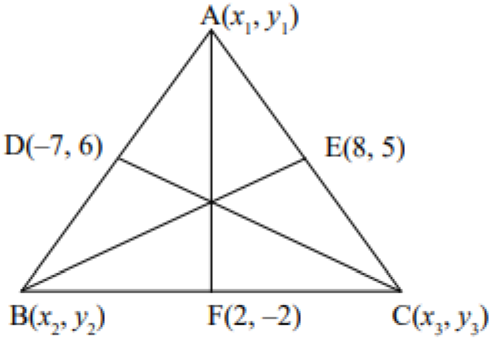
$\therefore \quad \angle A = \frac{1}{2} m(\text{arc BXD}) + \frac{1}{2} m(\text{arc BYD}) - m(\text{arc BXD})$ ...[From (ii) and (iii)]

$= \frac{1}{2} m(\text{arc BYD}) - \frac{1}{2} m(\text{arc BXD})$

$\therefore \quad \angle A = \frac{1}{2} [m(\text{arc BYD}) - m(\text{arc BXD})]$

(3) Find the co-ordinates of centroid of a triangle if points D(–7, 6), E(8, 5) and F(2, –2) are the midpoints of the sides of that triangle.

Solution:



Let $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$, $C \equiv (x_3, y_3)$

By the midpoint formula,

$$\frac{x_1 + x_2}{2} = -7$$

$\therefore \quad x_1 + x_2 = -14 \qquad$ ...(i)

$$\frac{y_1 + y_2}{2} = 6$$

$\therefore \quad y_1 + y_2 = 12 \qquad$ ...(ii)

$$\frac{x_2 + x_3}{2} = 2$$

$\therefore \quad x_2 + x_3 = 4 \qquad$ ...(iii)

$$\frac{y_2 + y_3}{2} = -2$$

$\therefore \quad y_2 + y_3 = -4 \qquad$ ...(iv)

$$\frac{x_1 + x_3}{2} = 8$$

$\therefore \quad x_1 + x_3 = 16 \qquad$ ...(v)

$$\frac{y_1 + y_3}{2} = 5$$

$\therefore \quad y_1 + y_3 = 10 \qquad$ ...(vi)

Adding equations (i), (iii) and (v),

$2x_1 + 2x_2 + 2x_3 = 6$

$\therefore \quad x_1 + x_2 + x_3 = 3$

$\therefore \quad \dfrac{x_1 + x_2 + x_3}{3} = \dfrac{3}{3}$

$\therefore \quad \dfrac{x_1 + x_2 + x_3}{3} = 1$

Adding equations (ii), (iv) and (vi),

$y_1 + y_2 + y_3 = 9$

$\therefore \quad \dfrac{y_1 + y_2 + y_3}{3} = 3$

But $G \equiv \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$ ...(Centroid formula)

$\therefore \quad G \equiv (1, 3)$

Q.5. Solve the following sub-question (any *one*): [3]

(1) *a* and *b* are natural numbers and *a* > *b*. If ($a^2 + b^2$), ($a^2 - b^2$) and 2*ab* are the sides of a triangle, then prove that the triangle is right angled.

Find out two Pythagorean triplets by taking suitable values of *a* and *b*.

Solution:

a and *b* are natural numbers and *a* > *b*. ...(Given)

Longest side = $(a^2 + b^2)$

Longest side = $(a^2 + b^2)$

$(a^2 + b^2)^2 = a^4 + 2a^2b^2 + b^4$ ...(i)

and

$(a^2 - b^2)^2 + (2ab)^2 = a^4 - 2a^2b^2 + b^4 + 4a^2b^2$

$\therefore \quad (a^2 - b^2)^2 + (2ab)^2 = a^4 + 2a^2b^2 + b^4$ ...(ii)

$\therefore \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$ ...[From (i) and (ii)]

$\therefore$ By converse of Pythagoras theorem,

$(a^2 + b^2)$, $(a^2 - b^2)$ and $2ab$ are the sides of a right-angled triangle.

Now, if $a = 2$ and $b = 1$ then

$a^2 + b^2 = 2^2 + 1^2 = 4 + 1 = 5$

$(a^2 - b^2) = 2^2 - 1^2 = 4 - 1 = 3$

$2ab = 2 \times 2 = 4$

∴ (3, 4, 5) is a Pythagorean triplet.

Similarly, if $a = 3$ and $b = 2$, then

$a^2 + b^2 = 3^2 + 2^2 = 9 + 4 = 13$

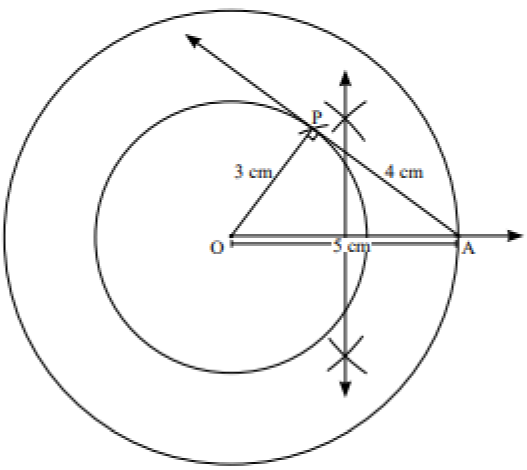
$a^2 - b^2 = 3^2 - 2^2 = 9 - 4 = 5$

$2ab = 2 \times 3 \times 2 = 12$

∴ (5, 12, 13) is a Pythagorean triplet.

(2) Construct two concentric circles with centre O and radii 3 cm and 5 cm. Construct a tangent to the smaller circle from any point A on the larger circle. Measure and wirte the length of the tangent segment. Calculate the length of the tangent segment using Pythagoras theorem.

Solution:



ΔOAP is a right angled triangle.

OA = 5 cm, OP = 3 cm, AP = ?

By Pythagoras theorem,

$AP^2 = OA^2 - OP^2$

$= 5^2 - 3^2$

$= 25 - 9$

$= 16$

∴ AP = 4

★★★