

Class 10 Real Numbers Important Questions and

Answers

Q.1: Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m .

Solution:

Let x be any positive integer and $y = 3$.

By Euclid's division algorithm;

$$x = 3q + r \text{ (for some integer } q \geq 0 \text{ and } r = 0, 1, 2 \text{ as } r \geq 0 \text{ and } r < 3)$$

Therefore,

$$x = 3q, 3q + 1 \text{ and } 3q + 2$$

As per the given question, if we take the square on both the sides, we get;

$$x^2 = (3q)^2 = 9q^2 = 3 \cdot 3q^2$$

$$\text{Let } 3q^2 = m$$

Therefore,

$$x^2 = 3m \dots\dots\dots(1)$$

$$x^2 = (3q + 1)^2$$

$$= (3q)^2 + 1^2 + 2 \times 3q \times 1$$

$$= 9q^2 + 1 + 6q$$

$$= 3(3q^2 + 2q) + 1$$

Substituting $3q^2 + 2q = m$ we get,

$$x^2 = 3m + 1 \dots\dots\dots (2)$$

$$x^2 = (3q + 2)^2$$

$$= (3q)^2 + 2^2 + 2 \times 3q \times 2$$

$$= 9q^2 + 4 + 12q$$

$$= 3(3q^2 + 4q + 1) + 1$$

Again, substituting $3q^2 + 4q + 1 = m$, we get,

$$x^2 = 3m + 1 \dots\dots\dots (3)$$

Hence, from eq. 1, 2 and 3, we conclude that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m .

Q.2: Express each number as a product of its prime factors:

(i) 140

(ii) 156

(iii) 3825

(iv) 5005

(v) 7429

Solution:

(i) 140

Using the division of a number by prime numbers method, we can get the product of prime factors of 140.

$$\text{Therefore, } 140 = 2 \times 2 \times 5 \times 7 \times 1 = 2^2 \times 5 \times 7$$

(ii) 156

Using the division of a number by prime numbers method, we can get the product of prime factors of 156.

$$\text{Hence, } 156 = 2 \times 2 \times 13 \times 3 = 2^2 \times 13 \times 3$$

(iii) 3825

Using the division of a number by prime numbers method, we can get the product of prime factors of 3825.

$$\text{Hence, } 3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$$

(iv) 5005

Using the division of a number by prime numbers method, we can get the product of prime factors of 5005.

$$\text{Hence, } 5005 = 5 \times 7 \times 11 \times 13 = 5 \times 7 \times 11 \times 13$$

(v) 7429

Using the division of a number by prime numbers method, we can get the product of prime factors of 7429.

Hence, $7429 = 17 \times 19 \times 23 = 17 \times 19 \times 23$

Q.3: Given that $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$.

Solution:

As we know that,

$\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$

So,

$$9 \times \text{LCM} = 306 \times 657$$

$$\text{LCM} = (306 \times 657)/9 = 22338$$

Therefore, $\text{LCM}(306, 657) = 22338$

Q.4: Prove that $3 + 2\sqrt{5}$ is irrational.

Solution:

Let $3 + 2\sqrt{5}$ be a rational number.

Then the co-primes x and y of the given rational number where ($y \neq 0$) is such that:

$$3 + 2\sqrt{5} = x/y$$

Rearranging, we get,

$$2\sqrt{5} = (x/y) - 3$$

$$\sqrt{5} = 1/2[(x/y) - 3]$$

Since x and y are integers, thus, $1/2[(x/y) - 3]$ is a rational number.

Therefore, $\sqrt{5}$ is also a rational number. But this confronts the fact that $\sqrt{5}$ is irrational.

Thus, our assumption that $3 + 2\sqrt{5}$ is a rational number is wrong.

Hence, $3 + 2\sqrt{5}$ is irrational.

Q.5: Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $13/3125$ (ii) $17/8$ (iii) $64/455$ (iv) $15/1600$

Solution:

Note: If the denominator has only factors of 2 and 5 or in the form of $2^m \times 5^n$ then it has a terminating decimal expansion.

If the denominator has factors other than 2 and 5 then it has a non-terminating repeating decimal expansion.

(i) $13/3125$

Factoring the denominator, we get,

$$3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$$

Or

$$= 2^0 \times 5^5$$

Since the denominator is of the form $2^m \times 5^n$ then, $13/3125$ has a terminating decimal expansion.

$$(ii) 17/8$$

Factoring the denominator, we get,

$$8 = 2 \times 2 \times 2 = 2^3$$

Or

$$= 2^3 \times 5^0$$

Since the denominator is of the form $2^m \times 5^n$ then, $17/8$ has a terminating decimal expansion.

$$(iii) 64/455$$

Factoring the denominator, we get,

$$455 = 5 \times 7 \times 13$$

Since the denominator is not in the form of $2^m \times 5^n$, therefore $64/455$ has a non-terminating repeating decimal expansion.

$$(iv) 15/1600$$

Factoring the denominator, we get,

$$1600 = 2^6 \times 5^2$$

Since the denominator is in the form of $2^m \times 5^n$, $15/1600$ has a terminating decimal expansion.

Q.6: The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p/q what can you say about the prime factors of q ?

(i) 43.123456789

(ii) 0.120120012000120000...

Solution:

(i) 43.123456789

Since it has a terminating decimal expansion, it is a rational number in the form of p/q and q has factors of 2 and 5 only.

(ii) 0.120120012000120000...

Since it has a non-terminating and non-repeating decimal expansion, it is an irrational number.

Q.7: Check whether 6^n can end with the digit 0 for any natural number n .

Solution:

If the number 6^n ends with the digit zero (0), then it should be divisible by 5, as we know any number with a unit place as 0 or 5 is divisible by 5.

Prime factorization of $6^n = (2 \times 3)^n$

Therefore, the prime factorization of 6^n doesn't contain the prime number 5.

Hence, it is clear that for any natural number n , 6^n is not divisible by 5 and thus it proves that 6^n cannot end with the digit 0 for any natural number n .

Q.8: What is the HCF of the smallest prime number and the smallest composite number?

Solution:

The smallest prime number = 2

The smallest composite number = 4

Prime factorisation of 2 = 2

Prime factorisation of 4 = 2×2

$$\text{HCF}(2, 4) = 2$$

Therefore, the HCF of the smallest prime number and the smallest composite number is 2.

Q.9: Using Euclid's Algorithm, find the HCF of 2048 and 960.

Solution:

$$2048 > 960$$

Using Euclid's division algorithm,

$$2048 = 960 \times 2 + 128$$

$$960 = 128 \times 7 + 64$$

$$128 = 64 \times 2 + 0$$

Therefore, the HCF of 2048 and 960 is 64.

Q.10: Find HCF and LCM of 404 and 96 and verify that $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$.

Solution:

Prime factorisation of 404 = $2 \times 2 \times 101$

Prime factorisation of 96 = $2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$$\text{HCF} = 2 \times 2 = 4$$

$$\text{LCM} = 2^5 \times 3 \times 101 = 9696$$

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

$$\text{Product of the given two numbers} = 404 \times 96 = 38784$$

Hence, verified that $\text{LCM} \times \text{HCF} = \text{Product of the given two numbers}$.

Important 2 Marks Questions for Class 10 Maths Board are as follows–

Q.1: Find the value of k for which the roots of the quadratic equation

, will have equal value.

Solution:

Given,

$$2x^2 + kx + 8 = 0$$

Comparing with the standard form $ax^2 + bx + c = 0$,

$$a = 2, b = k, c = 8$$

Condition for the equal roots is:

$$b^2 - 4ac = 0$$

$$k^2 - 4(2)(8) = 0$$

$$k^2 - 64 = 0$$

$$k^2 = 64$$

$$k = \pm 8$$

Q.2: Determine the AP whose third term is 5 and the seventh term is 9.

Solution:

Let a be the first term and d be the common difference of an AP.

Given,

Third term = 5

$$a + 2d = 5 \dots (i)$$

Seventh term = 9

$$a + 6d = 9 \dots (ii)$$

Subtracting (i) from (ii),

$$a + 6d - a - 2d = 9 - 5$$

$$4d = 4$$

$$d = 1$$

Substituting $d = 1$ in (i),

$$a + 2(1) = 5$$

$$a = 5 - 2 = 3$$

Therefore, the AP is: 3, 4, 5, 6,...

Q.3: Find a relation between x and y if the points $A(x, y)$, $B(-4, 6)$ and $C(-2, 3)$ are collinear.

Solution:

Let the given points be:

$$A(x, y) = (x_1, y_1)$$

$$B(-4, 6) = (x_2, y_2)$$

$$C(-2, 3) = (x_3, y_3)$$

If three points are collinear then the area of the triangle formed by these points is 0.

$$\text{i.e. } \left(\frac{1}{2}\right) |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$$

$$\left(\frac{1}{2}\right) |x(6 - 3) + (-4)(3 - y) + (-2)(y - 6)| = 0$$

$$x(3) - 4(3 - y) - 2(y - 6) = 0$$

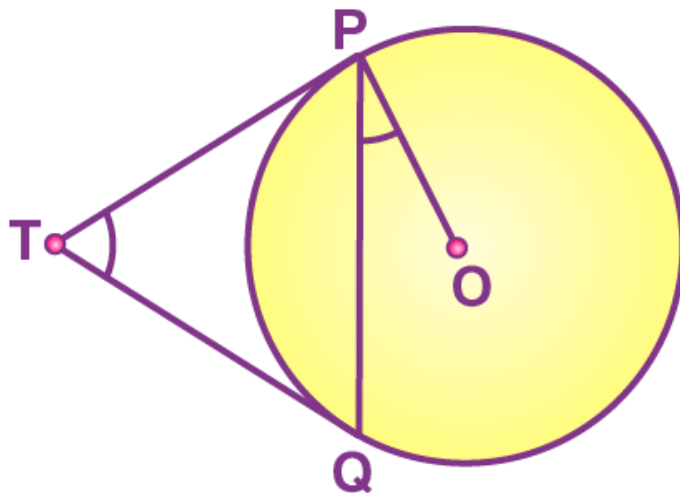
$$3x - 12 + 4y - 2y + 12 = 0$$

$$3x + 2y = 0$$

Or

$$x = -2y/3$$

Q.4: In the figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T, prove that $\angle PTQ = 2\angle OPQ$.



Solution:

Given that two tangents TP and TQ are drawn to a circle with centre O from an external point T

Let $\angle PTQ = \theta$.

Now, by using the theorem “the lengths of tangents drawn from an external point to a circle are equal”, we can say $TP = TQ$. So, TPQ is an isosceles triangle.

Thus,

$$\angle TPQ = \angle TQP = \frac{1}{2} (180^\circ - \theta) = 90^\circ - \left(\frac{1}{2}\right) \theta$$

By using the theorem, “the tangent at any point of a circle is perpendicular to the radius through the point of contact”, we can say $\angle OPT = 90^\circ$

Therefore,

$$\angle OPQ = \angle OPT - \angle TPQ = 90^\circ - \left[90^\circ - \left(\frac{1}{2}\right) \theta\right]$$

$$\angle OPQ = \left(\frac{1}{2}\right) \theta$$

$$\angle OPQ = \left(\frac{1}{2}\right) \angle PTQ$$

$$\Rightarrow \angle PTQ = 2 \angle OPQ.$$

Hence proved.

Q.5: A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. [Use $\pi = \frac{22}{7}$]

Solution:

Length of the arc = Length of wire

$$\left(\frac{\theta}{360^\circ}\right) 2\pi r = 22$$

$$\left(\frac{60^\circ}{360^\circ}\right) \times 2 \times \left(\frac{22}{7}\right) \times r = 22$$

$$\left(\frac{1}{6}\right) \times \left(\frac{2}{7}\right) \times r = 1$$

$$r = \left(\frac{7}{2}\right) \times 6$$

$$r = 21$$

Therefore, the radius of the circle is 21 cm.

Q.6: If a number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3, what is the probability that $x^2 \leq 4$?

Solution:

Sample space = $S = \{-3, -2, -1, 0, 1, 2, 3\}$

$$n(S) = 7$$

Let E be the event of choosing a number x such that $x^2 \leq 4$.

i.e. $x \leq \pm 2$

$$E = \{-2, -1, 0, 1, 2\}$$

$$n(E) = 5$$

$$P(E) = n(E)/n(S) = 5/7$$

Hence, the required probability is $5/7$.

Q.7: The larger of two supplementary angles exceeds the smaller by 18° . Find the angles.

Solution:

Let x and $(x + 18^\circ)$ be the supplementary angles.

That means,

$$x + (x + 18^\circ) = 180^\circ$$

$$2x = 180^\circ - 18^\circ$$

$$2x = 162^\circ$$

$$x = 162^\circ/2$$

$$x = 81^\circ$$

$$\text{Now, } x + 18^\circ = 81^\circ + 18^\circ = 99^\circ$$

Therefore, the supplementary angles are 81° and 99° .

Q.8: Find the mean of the following distribution:

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	5	10	10	7	8

Solution:

Class	Frequency (f_i)	Midpoint (x_i)	$f_i x_i$
3 – 5	5	4	20
5 – 7	10	6	60
7 – 9	10	8	80
9 – 11	7	10	70
11 – 13	8	12	96
Total	$\Sigma f_i = 40$	$\Sigma x_i = 40$	$\Sigma f_i x_i = 326$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= 326/40$$

$$= 8.15$$

Therefore, the mean of the given distribution is 8.15.

Q.9: Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

Solution:

Let $(5 + 3\sqrt{2})$ be a rational number.

$$5 + 3\sqrt{2} = p/q \text{ (Where } q \neq 0 \text{ and } p \text{ and } q \text{ are co-prime numbers)}$$

$$3\sqrt{2} = (p/q) - 5$$

$$3\sqrt{2} = (p - 5q)/q$$

$$\sqrt{2} = (p - 5q)/3q$$

p and q are integers and $q \neq 0$

Thus, $(p - 5q)/3q$ is rational number.

Also, $\sqrt{2}$ is a rational number.

However, it is given that $\sqrt{2}$ is an irrational number.

This is a contradiction, and thus, our assumption that $(5 + 3\sqrt{2})$ be a rational number is wrong.

That means $(5 + 3\sqrt{2})$ is an irrational number.

Hence proved.

Q.10: Find the value of p , for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

Solution:

Given quadratic equation is:

$$px^2 - 14x + 8 = 0$$

Let α and 6α be the roots of the given quadratic equation.

Sum of the roots = $-\text{coefficient of } x / \text{coefficient of } x^2$

$$\alpha + 6\alpha = -(-14)/p$$

$$7\alpha = 14/p$$

$$\alpha = 2/p \dots (i)$$

Product of roots = $\text{constant term} / \text{coefficient of } x^2$

$$(\alpha)(6\alpha) = 8/p$$

$$6\alpha^2 = 8/p$$

Substituting $\alpha = 2/p$ from (i),

$$6 \times (2/p)^2 = 8/p$$

$$24/p^2 = 8/p$$

$$3/p = 1$$

$$p = 3$$

Therefore, the value of p is 3.

Important 4 Marks Questions and Solutions for Class 12 Maths Board are as follows-

Question 1- If m th term of an A.P. is

and n th term is

, then find the sum of its first mn terms.

Solution:

Let a be the first term and d be the common difference of an AP.

Given,

$$a_m = 1/n$$

$$\text{i.e. } a + (m - 1)d = 1/n$$

$$n[a + (m - 1)d] = 1$$

$$an + mnd - nd = 1 \dots (i)$$

And

$$a_n = 1/m$$

$$a + (n - 1)d = 1/m$$

$$m[a + (n - 1)d] = 1$$

$$am + mnd - md = 1 \dots (ii)$$

From (i) and (ii),

$$an + mnd - nd = am + mnd - md$$

$$an - am = nd - md$$

$$a(n - m) = (n - m)d$$

$$a = d$$

Substituting $a = d$ in equation (i),

$$dn + mnd - nd = 1$$

$$mnd = 1$$

$$d = 1/mn$$

Therefore, $a = 1/mn$

Sum of first mn terms is:

$$S_{mn} = (mn/2)[2(1/mn) + (mn - 1)(1/mn)]$$

$$= (mn/2)[(2/mn) + (mn - 1)/mn]$$

$$= (mn/2mn)[2 + mn - 1]$$

$$= (1/2)(mn + 1)$$

Hence, the sum of first mn terms $= (mn + 1)/2$.

Question 2- A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from

to

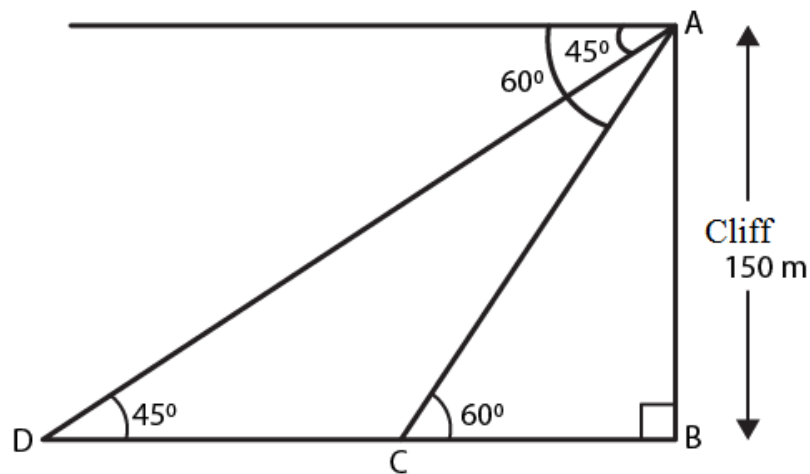
in 2 minutes. Find the speed of the boat in m/h.

Solution:

Let AB be the height of the cliff.

$$AB = 150 \text{ m}$$

C and D be the position of a boat.



Boat is moved from C to D in 2 minutes = $\frac{2}{60}$ hr = $\frac{1}{30}$ hr

In right triangle ABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{150}{BC}$$

$$BC = \frac{150}{\sqrt{3}} \text{ m}$$

In right triangle ABD,

$$\tan 45^\circ = \frac{AB}{AD}$$

$$1 = 150/(BC + CD)$$

$$(150/\sqrt{3}) + CD = 150$$

$$CD = 150 - (150/\sqrt{3})$$

$$CD = (150\sqrt{3} - 150)/\sqrt{3}$$

$$= 150(\sqrt{3} - 1)/\sqrt{3} \text{ m}$$

$$\text{Distance CD} = \text{Speed} \times \text{time}$$

$$150(\sqrt{3} - 1)/\sqrt{3} = \text{Speed} \times (1/30)$$

$$\text{Speed} = 30 \times [150(\sqrt{3} - 1)/\sqrt{3}]$$

$$= 1500\sqrt{3}(\sqrt{3} - 1) \text{ m/h}$$

Question 3- Two different dice are thrown together. Find the probability that the numbers obtained

(i) have a sum less than 7

(ii) have a product less than 16

(iii) is a doublet of odd numbers.

Solution:

Given,

Two different dice are thrown together.

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$$\text{Total number of outcomes} = n(S) = 36$$

(i) have a sum less than 7

Let E_1 be the event of getting the numbers on dice whose sum is less than 7.

$$E_1 = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)\}$$

$$n(E_1) = 15$$

$$P(E_1) = n(E_1)/n(S) = 15/36 = 5/12$$

(ii) have a product less than 16

Let E_2 be the event of getting the numbers on dice whose product is less than 16.

$$E_2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2)\}$$

$$n(E_2) = 25$$

$$P(E_2) = n(E_2)/n(S) = 25/36$$

(iii) is a doublet of odd numbers.

Let E_3 be the event of getting a doublet of odd numbers.

$$E_3 = \{(1, 1), (3, 3), (5, 5)\}$$

$$P(E_3) = n(E_3)/n(S) = 3/36 = 1/12$$

Question 4- The area of a triangle is 5 sq units. Two of its vertices are (2,1) and (3,-2). If the third vertex is

, then find the value of y.

Solution:

Let A, B, and C be the vertices of a triangle.

$$A(2, 1) = (x_1, y_1)$$

$$B(3, -2) = (x_2, y_2)$$

$$C(7/2, y) = (x_3, y_3)$$

Given,

Area of the triangle is 5 sq.units.

$$(1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 5$$

$$(1/2)|2(-2 - y) + 3(y - 1) + (7/2)(1 + 2)| = 5$$

$$|-4 - 2y + 3y - 3 + (7/2)(3)| = 10$$

$$y - 7 + (21/2) = 10$$

$$y = 10 + 7 - (21/2)$$

$$y = 17 - (21/2)$$

$$y = (34 - 21)/2$$

$$y = 13/2$$

Therefore, the value of y is $13/2$.

Question 5- The

th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with an internal radius of 10 cm. Find the height of water in the cylindrical vessel.

Solution:

Given,

Height of conical vessel = $h = 24$ cm

Radius of conical vessel = $r = 5$ cm

Volume of water = $(3/4) \times$ Volume of conical vessel

$$= (3/4) \times (1/3)\pi r^2 h$$

$$= (3/4) \times (1/3) \times (\pi \times 5 \times 5 \times 24)$$

$$= 150\pi \text{ cm}^3$$

Let H be the height of a cylindrical vessel.

Radius of cylindrical vessel = R = 10 cm {given}

From the given,

Volume of cylindrical vessel = Volume of water

$$\pi R^2 H = 150\pi$$

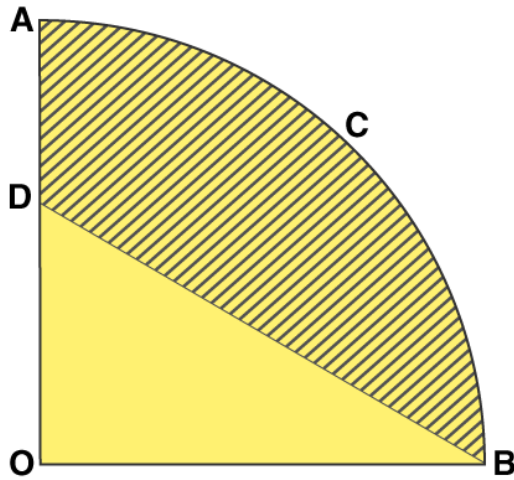
$$R^2 H = 150$$

$$10 \times 10 \times H = 150$$

$$H = 1.5 \text{ cm}$$

Therefore, the height of a cylindrical vessel.

Question 6- In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the shaded region.



Solution:

Given,

Radius of the quadrant (r) = 3.5 cm

$$\text{Area of quadrant} = \left(\frac{1}{4}\right)\pi r^2$$

$$= \left(\frac{1}{4}\right) \times \left(\frac{22}{7}\right) \times 3.5 \times 3.5$$

$$= 9.625 \text{ cm}^2$$

$$\text{Area of triangle BOD} = \left(\frac{1}{2}\right) \times OB \times OD$$

$$= \left(\frac{1}{2}\right) \times 3.5 \times 2$$

$$= 3.5 \text{ cm}^2$$

$$\text{Area of the shaded region} = \text{Area of quadrant} - \text{Area of triangle BOD}$$

$$= 9.625 - 3.5$$

$$= 6.125 \text{ cm}^2$$

Question 7- If the equation

has equal roots then show that

.

Solution:

Given,

$$(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$$

Comparing with the standard form of a quadratic equation $ax^2 + bx + c = 0$,

$$a = (1 + m^2)$$

$$b = 2mc$$

$$c = (c^2 - a^2)$$

Given that the quadratic equation has equal roots.

So, $b^2 - 4ac = 0$ {determinant is equal to 0}

$$(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$

$$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$

$$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$$

$$4c^2 = 4a^2m^2 + 4a^2$$

$$4c^2 = 4a^2(m^2 + 1)$$

$$c^2 = a^2(1 + m^2)$$

Question 8- Find the sum of n terms of the series

Solution:

Given series:

$$[4 - (1/n)] + [4 - (2/n)] + [4 - (3/n)] + \dots \text{ up to } n \text{ terms}$$

$$= (4 + 4 + 4 + \dots \text{ up to } n \text{ terms}) - [(1/n) + (2/n) + (3/n) + \dots \text{ up to } n \text{ terms}]$$

$$= 4n - (1/n)[1 + 2 + 3 + \dots \text{ up to } n \text{ terms}]$$

$$= 4n - (1/n)[n(n + 1)/2] \quad \{\text{sum of first } n \text{ natural numbers} = n(n + 1)/2\}$$

$$= 4n - (n + 1)/2$$

$$= (8n - n - 1)/2$$

$$= (7n - 1)/2$$

Therefore, the sum of the given series is $(7n - 1)/2$.

Question 9- Prove that: $(\sec A - \cos A) \cdot (\cot A + \tan A) = \tan A \cdot \sec A$

Solution:

$$\text{LHS} = (\sec A - \cos A)(\cot A + \tan A)$$

$$= [(1/\cos A) - \cos A] [(\cos A/\sin A) + (\sin A/\cos A)]$$

$$= [(1 - \cos^2 A)/\cos A] [(\cos^2 A + \sin^2 A)/(\sin A \cos A)]$$

$$= (\sin^2 A / \cos A) [1/(\sin A \cos A)]$$

$$= \sin A / (\cos A \cos A)$$

$$= (\sin A / \cos A) (1/\cos A)$$

$$= \tan A \sec A$$

$$= \text{RHS}$$

$$\text{Thus, } (\sec A - \cos A) \cdot (\cot A + \tan A) = \tan A \cdot \sec A$$

Hence proved.

Question 10- The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f.

Daily pocket allowance (in Rs.)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of children	7	6	9	13	f	5	4

Solution:

Class interval	Number of children (f_i)	Mid-point (x_i)	$f_i x_i$
11 – 13	7	12	84

13 – 15	6	14	84
15 – 17	9	16	144
17 – 19	13	18	234
19 – 21	f	20	20f
21 – 23	5	22	110
23 – 25	4	24	96
Total	$\sum f_i = 44 + f$		$\sum f_i x_i = 752 + 20f$

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{(752 + 20f)}{(44 + f)}$$

$$\text{Given that, } \frac{(752 + 20f)}{(44 + f)} = 18$$

$$752 + 20f = 18(44 + f)$$

$$752 + 20f = 792 + 18f$$

$$20f - 18f = 792 - 752$$

$$2f = 40$$

$$f = 20$$

Therefore, the missing frequency = $f = 20$

Important 4 Marks Questions and Solutions for Class 10 Maths Board are as follows-

Question 1- Solve for x:

Solution:

Taking LCM and cross multiplying the terms,

$$x[x - 1 + 2x - 4] = 6[x^2 - 2x - x + 2]$$

$$x(3x - 5) = 6(x^2 - 3x + 2)$$

$$6x^2 - 18x + 12 - 3x^2 + 5x = 0$$

$$3x^2 - 13x + 12 = 0$$

$$3x^2 - 9x - 4x + 12 = 0$$

$$3x(x - 3) - 4(x - 3) = 0$$

$$(3x - 4)(x - 3) = 0$$

$$3x - 4 = 0, x - 3 = 0$$

$$x = 4/3, 3$$

Question 2-

Find the sum of first 24 terms of an A.P. whose nth term is given by

.

Solution:

Given,

$$a_n = 3 + 2n$$

Substituting $n = 1, 2, 3, \dots$

When $n = 1$,

$$a_1 = 3 + 2(1) = 3 + 2 = 5$$

When $n = 2$,

$$a_2 = 3 + 2(2) = 3 + 4 = 7$$

When $n = 3$,

$$a_3 = 3 + 2(3) = 3 + 6 = 9$$

Thus, the AP is: 5, 7, 9,...

Here,

First term = $a = 5$

Common difference = $d = 2$

We know that,

Sum of first n terms of an AP = $S_n = \frac{n}{2} [2a + (n - 1)d]$

Now,

$$S_{24} = (24/2) [2(5) + (24 - 1)(2)]$$

$$= 12[10 + 23(2)]$$

$$= 12[10 + 46]$$

$$= 12 \times 56$$

$$= 672$$

Therefore, the sum of the first 24 terms of the given AP is 672.

Question 3-

A bucket, is in the form of a frustum of a cone whose height is 42 cm and the radii of its circular ends are 30 cm and 10 cm. Find the amount of milk (in litres) which this bucket can hold. If the milkman sells the milk at the rate of Rs. 40 per litre, what amount he will get from the sale?

If the milkman sells half the milk at less rate to the economically weaker section of society, what value he exhibits by doing this?

Solution:

Given that a bucket is in the form of a frustum of a cone with the following dimensions.

Height = $h = 42$ cm

$R = 30$ cm

$$r = 10 \text{ cm}$$

$$\text{Volume of frustum of a cone} = \left(\frac{1}{3}\right)\pi h[R^2 + r^2 + Rr]$$

Substituting the values,

$$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 42 \times [(30)^2 + (10)^2 + (30)(10)]$$

$$= 44 \times [900 + 100 + 300]$$

$$= 44 \times 1300$$

$$= 572000 \text{ cm}^3$$

$$= (572000/1000) \text{ litres} \{\text{since } 1 \text{ cm}^3 = 1/1000 \text{ litres}\}$$

$$= 57.2 \text{ litres}$$

Therefore, the bucket can hold 57.2 litres of milk.

Also,

$$\text{Cost of 1 litre milk} = \text{Rs. } 40 \text{ (given)}$$

$$\text{So, the cost of 57.2 litres of milk} = 57.2 \times \text{Rs. } 40 = \text{Rs. } 2288$$

Hence, the milkman can get Rs. 2288 from selling the milk in the bucket.

If the milkman sells half the milk at less rate to the economically weaker section of society, he shows the values of help, kindness, and empathy.

Question 4-

If the coordinates of two points are A(3,4), B (5,-2) and a point P (x,5) is such that PA = PB, then find the area of

.

Solution:

Given points:

A(3, 4), B(5, -2) and P(x, 5)

Also,

$$PA = PB$$

Squaring on both sides,

$$PA^2 = PB^2$$

Using distance formula, $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$;

$$(x - 3)^2 + (5 - 4)^2 = (x - 5)^2 + (5 + 2)^2$$

$$x^2 + 9 - 6x + 1 = x^2 + 25 - 10x + 49$$

$$10 - 6x = 74 - 10x$$

$$\Rightarrow 10x - 6x = 74 - 10$$

$$\Rightarrow 4x = 64$$

$$\Rightarrow x = 16$$

$$\Rightarrow P = (x, 5) = (16, 5)$$

Let,

$$P(16, 5) = (x_1, y_1), A(3, 4) = (x_2, y_2), B(5, -2) = (x_3, y_3)$$

$$\text{Area of triangle PAB} = (1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= (1/2)|16(4 + 2) + 3(-2 - 5) + 5(5 - 4)|$$

$$= (1/2)|16(6) + 3(-7) + 5(1)|$$

$$= (1/2)|96 - 21 + 5|$$

$$= (1/2) \times 80$$

$$= 40 \text{ square units}$$

Question 5-

A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys so formed.

Solution:

Given,

Height of cylinder = H = 15 cm

Diameter = 12 cm

Radius of cylinder = $R = 12/2 = 6$ cm

Radius of cone = 3 cm

Height of cone = $h = 9$ cm

Let n be the number of toys in the shape of a cone made.

So,

Volume of cylinder = $n \times$ volume of a cone

$$\pi R^2 H = (1/3) \pi r^2 h$$

$$6 \times 6 \times 15 = n \times (1/3) \times 3 \times 3 \times 9$$

$$n = (6 \times 6 \times 15 \times 3) / (3 \times 3 \times 9)$$

$$n = 20$$

Question 6-

From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed. The remaining cards are mixed and a card is drawn at random. Find the probability that the drawn card is

(i) a black Queen

(ii) a card of red colour

(iii) a Jack of black colour

(iv) a face card

Solution:

We know that, $P(\text{an event}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

Given,

From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed.

So, the total number of outcomes = $52 - (2 + 2 + 2 + 2) = 52 - 8 = 44$

(i) a black Queen

Number of black queen cards in the available cards = 0

$P(\text{getting a black queen}) = \frac{0}{44} = 0$

(ii) a card of red colour

Number of red colour cards = $26 - 4 = 22$

$P(\text{getting a card of red colour}) = \frac{22}{44} = \frac{1}{2}$

(iii) a Jack of black colour

Number of black Jack cards = 2

$P(\text{getting a Jack of black colour}) = \frac{2}{44} = \frac{1}{22}$

(iv) a face card

$$\text{Number of face cards} = 12 - 6 = 6$$

$$P(\text{getting a face card}) = 6/44 = 3/22$$

Question 7-

From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some vertically over the top the building is

and

respectively. Find the height of the helicopter above the ground.

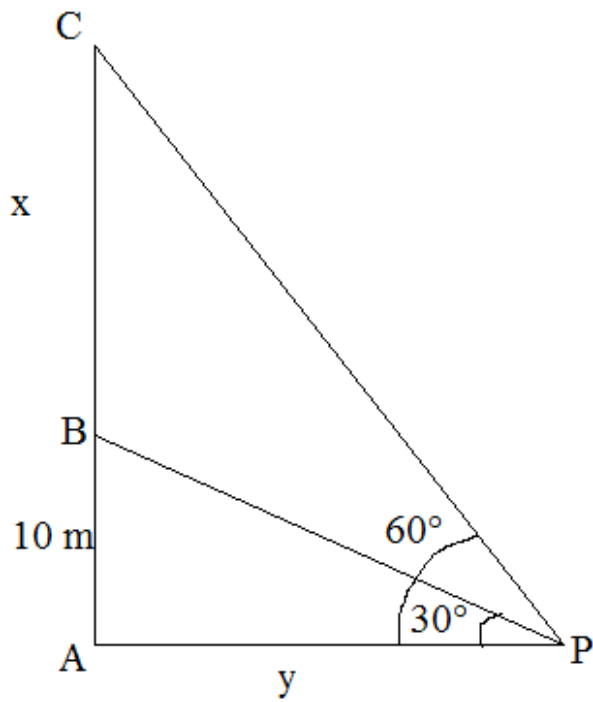
Solution:

Let P be the position of a point and C be the position of a helicopter.

Height of the building AB = 10 m

$$AC = x \text{ m}$$

$$PA = y \text{ m}$$



In right triangle PAB,

$$\tan 30^\circ = AB/PA$$

$$1/\sqrt{3} = 10/y$$

$$y = 10\sqrt{3} \text{ m}$$

In right triangle PAC,

$$\tan 60^\circ = AC/PA$$

$$\sqrt{3} = (x + 10)/10\sqrt{3}$$

$$10\sqrt{3} \sqrt{3} = x + 10$$

$$x = 30 - 10 = 20$$

Therefore, the height of helicopter from the ground = $x + 10 = 20 + 10 = 30$ m

Question 8-

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm.

Construct tangents to each circle from the centre of the other circle.

Solution:

Steps of Construction:

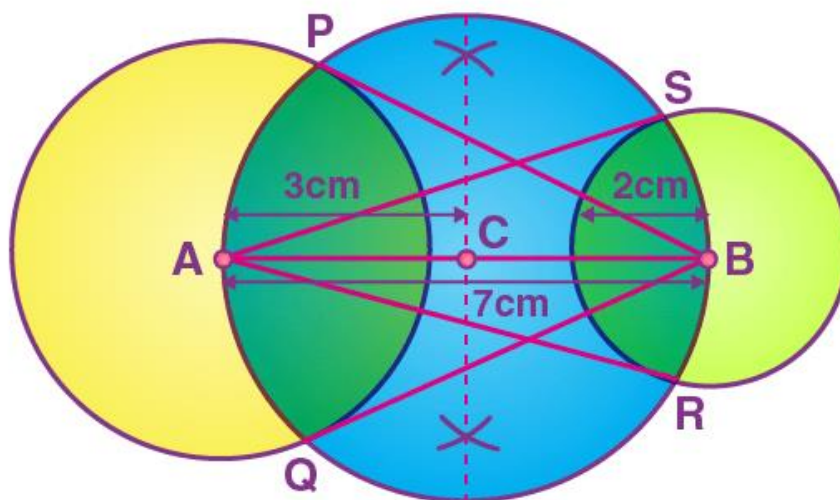
Step 1: Draw a line segment AB of 7 cm.

Step 2: Taking A and B as centres, draw two circles of 3 cm and 2 cm radius respectively.

Step 3: Bisect the line AB. Let the midpoint of AB be C.

Step 4: Taking C as centre, draw a circle of radius AC which intersects the two circles at point P, Q, R and S.

Step 5: Join BP, BQ, AS and AR.



Here, BP, BQ, AS and AR are the tangents to the circles.

Question 9-

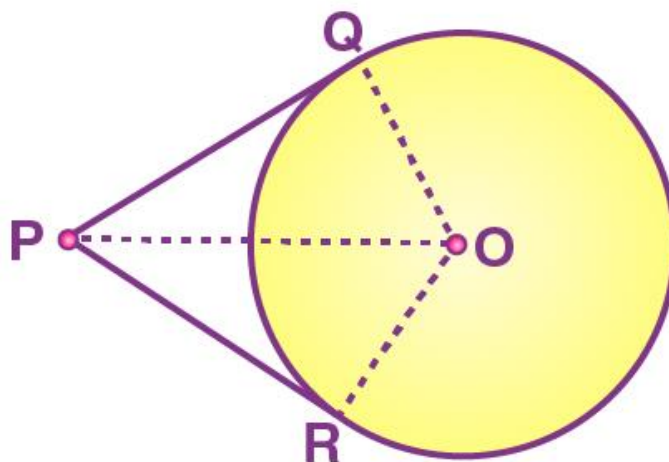
Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:

Consider a circle with the centre "O" and P is the point that lies outside the circle. Hence, the two tangents formed are PQ and PR.

We need to prove: $PQ = PR$.

To prove the tangent PQ is equal to PR, join OP, OQ and OR. Hence, $\angle OQP$ and $\angle ORP$ are the right angles.



Therefore, $OQ = OR$ (Radii)

$OP = OP$ (Common side)

By using the RHS rule, we can say, $\triangle OQP \cong \triangle ORP$.

Thus, by using the CPCT rule, the tangent $PQ = PR$.

Hence proved.

Question 10-

The Life insurance agent found the following data for the distribution of ages of 100 policyholders. Calculate the median age, if policies are given only to the persons whose age is 18 years onwards but less than the 60 years.

Age (in years)	Number of policyholders
----------------	-------------------------

Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Solution:

Let us calculate the class intervals and the corresponding frequencies for the given data.

Class interval	Frequency	Cumulative frequency
15 – 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45

35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100

$$N = 100$$

$$N/2 = 100/2 = 50$$

The cumulative frequency greater than and nearer to 50 is 78 which belongs to the class interval 35 – 40.

So, the median class is 35 – 40.

Frequency of the median class = $f = 33$

Lower limit of median class = $l = 35$

Cumulative frequency of the class preceding the median class = $cf = 45$

Class size = $h = 5$

$$\text{Median} = l + [(N/2 - cf)/f] \times h$$

$$= 35 + [(50 - 45)/33] \times 5$$

$$= 35 + (25/33)$$

$$= 35 + 0.75$$

$$= 35.75 \text{ (approx.)}$$

Therefore, the median age = 35.75 years.