Q.1: How many tangents can be drawn from the external point to a circle?

Answer: Two tangents can be drawn from the external point to a circle.

Q.2: Given: A triangle OAB which is an isosceles triangle and AB is tangent to the circle with centre O. Find the measure of ∠OAB.

Answer: The measure of ∠OAB in the given isosceles triangle OAB will be 45 degrees.

Q.3: What should be the angle between the two tangents which are drawn at the end of two radii and are inclined at an angle of 45 degrees?

Answer: The angle between them shall be 135 degrees.

Q.4: Given a right triangle PQR which is right-angled at Q. QR = 12 cm, PQ = 5 cm. The radius of the circle which is inscribed in triangle PQR will be?

Answer: The radius of the circle will be 2 cm.

Q.5: Define Tangent and Secant.

Answer: A tangent is a line which meets the circle only at one point.

A secant is a line which meets the circle at two points while intersecting it. These two points are always distinct.

Q.6: What is a circle?

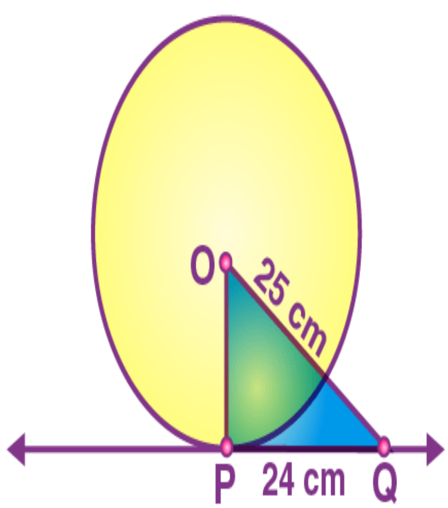
Answer: If we collect all the points given on a plane and are at a constant distance, we will get a circle. The constant distance is the radius and the fixed point will be the centre of the circle.

Long Answer Type Questions

Q.1: From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

Solution:

First, draw a perpendicular from the center O of the triangle to a point P on the circle which is touching the tangent. This line will be perpendicular to the tangent of the circle.



So, OP is perpendicular to PQ i.e. OP ⊥ PQ

From the above figure, it is also seen that △OPQ is a right-angled triangle.

It is given that

OQ = 25 cm and PQ = 24 cm

By using Pythagorean theorem in △OPQ,

$OQ^2 = OP^2 + PQ^2$

=> $(25)^2 = OP^2 + (24)^2$

=> $OP^2 = 625 – 576$

=> $OP^2 = 49$

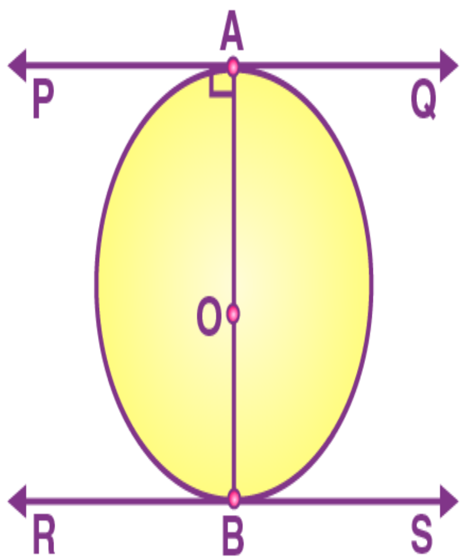
=> OP = 7 cm

Therefore, the radius of the given circle is 7 cm.

Q. 2: Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:

First, draw a circle and connect two points A and B such that AB becomes the diameter of the circle. Now, draw two tangents PQ and RS at points A and B respectively.



Now, both radii i.e. AO and OB are perpendicular to the tangents.

So, OB is perpendicular to RS and OA perpendicular to PQ

So, ∠OAP = ∠OAQ = ∠OBR = ∠OBS = 90°

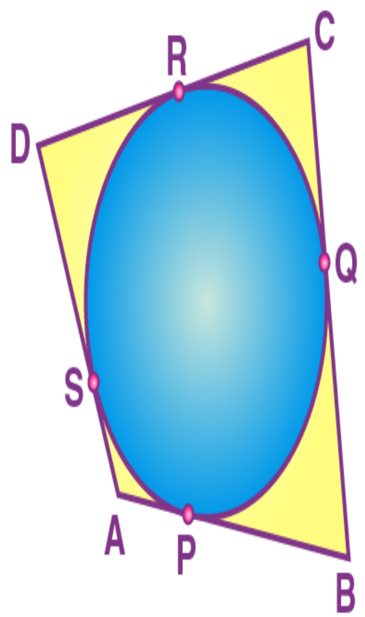
From the above figure, angles OBR and OAQ are alternate interior angles.

Also, ∠OBR = ∠OAQ and ∠OBS = ∠OAP {since they are also alternate interior angles}

So, it can be said that line PQ and the line RS will be parallel to each other.

Hence Proved.

Q. 3: A quadrilateral ABCD is drawn to circumscribe a circle as shown in the figure. Prove that AB + CD = AD + BC



Solution:

From this figure,

(i) DR = DS

(ii) BP = BQ

(iii) AP = AS

(iv) CR = CQ

Since they are tangents on the circle from points D, B, A, and C respectively.

Now, adding the LHS and RHS of the above equations we get,

DR + BP + AP+ CR = DS + BQ + AS + CQ

By rearranging them we get,

(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)

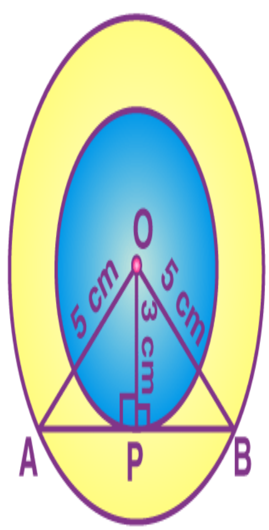
By simplifying,

AD + BC = CD + AB

Q. 4: Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

Draw two concentric circles with the centre O. Now, draw a chord AB in the larger circle which touches the smaller circle at a point P as shown in the figure below.



From the above diagram, AB is tangent to the smaller circle to point P.

∴ OP ⊥ AB

Using Pythagoras theorem in triangle OPA,

$OA^2 = AP^2 + OP^2$

=> $5^2 = AP^2 + 3^2$

=> $AP^2 = 25 – 9 = 16$

=> AP = 4

OP ⊥ AB

Since the perpendicular from the centre of the circle bisects the chord, AP will be equal to PB

So, AB = 2AP = 2 × 4 = 8 cm

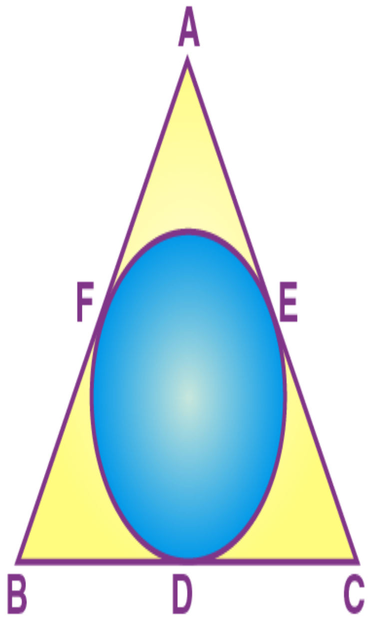
Hence, the length of the chord of the larger circle is 8 cm.

Q. 5: Let s denote the semi-perimeter of a triangle ABC in which BC = a, CA = b, AB = c. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that BD = s – b.

Solution:

According to the question,

A triangle ABC with BC = a , CA = b and AB = c . Also, a circle is inscribed which touches the sides BC, CA and AB at D, E and F respectively and s is semi perimeter of the triangle



To Prove: BD = s − b

Proof:

According to the question,

We have,

Semi Perimeter = s

Perimeter = 2s

2s = AB + BC + AC…. [1]

As we know,

Tangents drawn from an external point to a circle are equal

So we have

AF = AE… [2] [Tangents from point A]

BF = BD ….[3] [Tangents From point B]

CD = CE…. [4] [Tangents From point C]

Adding [2], [3], and [4],

AF + BF + CD = AE + BD + CE

AB + CD = AC + BD

Adding BD both side,

AB + CD + BD = AC + BD + BD

AB + BC – AC = 2BD

AB + BC + AC – AC – AC = 2BD

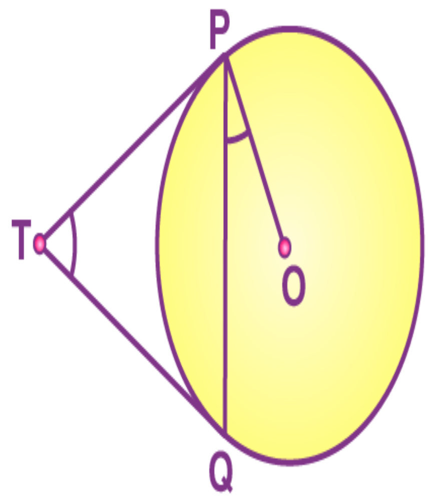
2s – 2AC = 2BD [From (1)]

2BD = 2s – 2b [as AC = b]

BD = s – b

Hence proved.

Q.6: In the figure, two tangents TP and TQ are drawn to a circle with center O from an external point T, prove that ∠PTQ = 2OPQ.



Solution:

Given that two tangents TP and TQ are drawn to a circle with centre O from an external point T

Let ∠PTQ = θ.

Now, by using the theorem "the lengths of tangents drawn from an external point to a circle are equal", we can say TP = TQ. So, TPQ is an isosceles triangle.

Thus,

∠TPQ = ∠TQP = ½ (180°- θ) = 90° – (½) θ

By using the theorem, "the tangent at any point of a circle is perpendicular to the radius through the point of contact", we can say ∠OPT = 90°

Therefore,

$\angle OPQ = \angle OPT - \angle TPQ = 90^\circ - [90^\circ - (½)\,\theta]$

$\angle OPQ = (½)\theta$

$\angle OPQ = (½)\ \angle PTQ$

$\Rightarrow \angle PTQ = 2\ \angle OPQ$.

Hence proved.

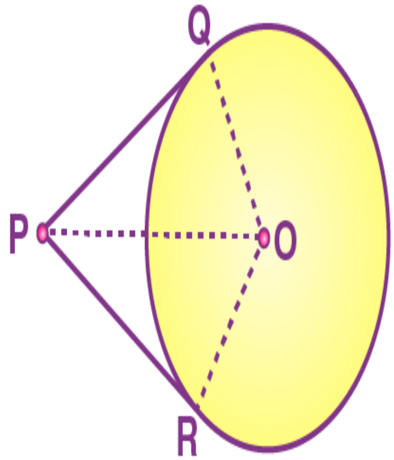
Q.7: Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:

Consider a circle with the centre “O” and P is the point that lies outside the circle. Hence, the two tangents formed are PQ and PR.

We need to prove: PQ = PR.

To prove the tangent PQ is equal to PR, join OP, OQ and OR. Hence, ∠OQP and ∠ORP are the right angles.



Therefore, OQ = OR (Radii)

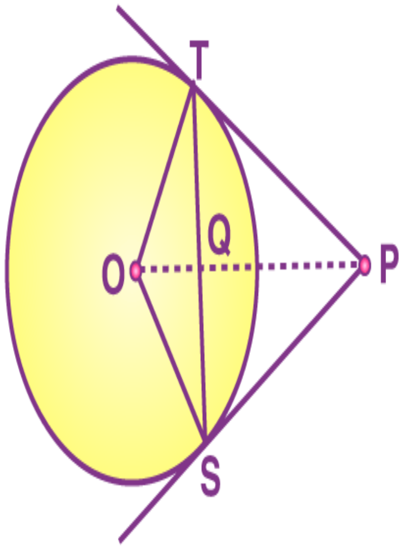
OP = OP (Common side)

By using the RHS rule, we can say, Δ OQP ≅ Δ ORP.

Thus, by using the CPCT rule, the tangent PQ = PR.

Hence proved.

Q.8: In the figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that ∠OTS = ∠OST = 30°.



Solution:

Given that from an external point P, Two tangents PT and PS are drawn to a circle with center O and radius r and OP = 2r

OS = OT {radii of same circle}

∠OTS = ∠OST {angles opposite to equal sides are equal} ….(i)

A tangent drawn at a point on a circle is perpendicular to the radius through point of contact.

OT ⊥ TP and OS ⊥ SP

∠OSP = 90°

∠OST + ∠PST = 90°

$\angle PST = 90^\circ - \angle OST$....(ii)

In triangle PTS

PT = PS {tangents drawn from an external point to a circle are equal}

$\angle PST = \angle PTS = 90^\circ - \angle OST$ {from (ii)}

In $\triangle PTS$

$\angle PTS + \angle PST + \angle SPT = 180^\circ$ {angle sum property of a triangle}

$90^\circ - \angle OST + 90^\circ - \angle OST + \angle SPT = 180^\circ$

$\angle SPT = 2\angle OST$....(iii)

In $\triangle OTP$, $OT \perp TP$

$\sin(\angle OPT) = OT/OP = r/2r = 1/2$

$\sin(\angle OPT) = \sin 30^\circ$

$\angle OPT = 30^\circ$....(iv)

Similarly,

In $\triangle OSP$,

$\angle OPS = 30^\circ$....(v)

Adding (iv) and (v),

$\angle OPT + \angle OPS = 30^\circ + 30^\circ$

∠SPT = 60°

Now substituting this value in (iii),

∠SPT = 2∠OST

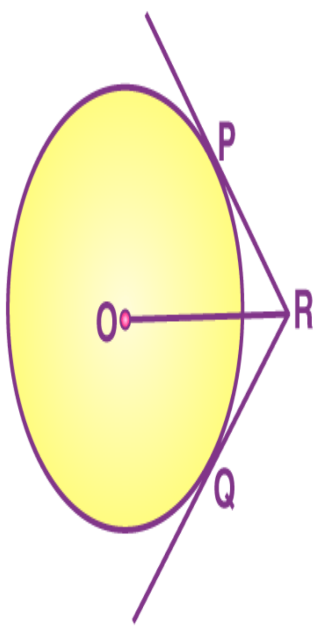
60° = 2∠OST

∠OST = 30°….(vi)

From (i) and (vi),

∠OST = ∠OTS = 30°

Q.9: In the figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If ∠PRQ = 120°, then prove that OR = PR + RQ.



Solution:

Given, two tangents RQ and RP are drawn from an external point R to the circle with centre O. ∠PRQ = 120°

Join OP, OQ and OR.

∠PRQ = ∠QRO = 120°/2 = 60°

RQ and RP are the tangent to the circle.

OQ and OP are radii

OQ ⊥ QR and OP ⊥ PR

Form right ΔOPR,

∠POR = 180° – (90° + 60°) = 30°

and ∠QOR = 30°

cos a = PR/OR (suppose 'a' be the angle)

cos 60° =PR/OR

1/2 = PR/OR

OR = 2 PR

Again from right ΔOQR,

OR = 2 QR

From both the results, we have

2 PR + 2 QR = 2OR

or OR = PR + RQ

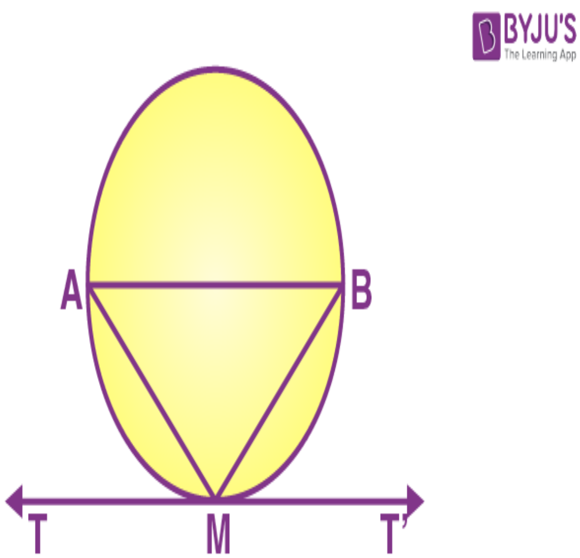
Hence Proved.

Q.10: Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the endpoints of the arc.

Solution:

Let mid-point of an arc AMB be M and TMT’ be the tangent to the circle.

Now, join AB, AM and MB.



Since, arc AM = arc MB

⇒ Chord AM = Chord MB

In $\triangle AMB$, $AM = MB$

⇒ $\angle MAB = \angle MBA$….(i) {angles corresponding to the equal sides are equal}

Since, TMT’ is a tangent line.

$\angle AMT = \angle MBA$ {angles in alternate segment are equal}

Thus, $\angle AMT = \angle MAB$ {from (i)}

But $\angle AMT$ and $\angle MAB$ are alternate angles, which is possible only when AB || TMT’

Therefore, the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the endpoints of the arc.

Hence proved.