Class 10 Math's Chapter 2 Important Questions With Solutions

A few important Class 10 polynomials questions are provided below with solutions. These questions include both short and long answer questions to let the students get acquainted with the in-depth concepts.

Q.1: Find the value of "p" from the polynomial $x^2 + 3x + p$, if one of the zeroes of the polynomial is 2.

Solution:

As 2 is the zero of the polynomial.

We know that if α is a zero of the polynomial p(x), then $p(\alpha) = 0$

Substituting x = 2 in $x^2 + 3x + p$,

$\Rightarrow 2^2 + 3(2) + p = 0$

$\Rightarrow 4 + 6 + p = 0$

$\Rightarrow 10 + p = 0$

$\Rightarrow p = -10$

Q.2: Does the polynomial $a^4 + 4a^2 + 5$ have real zeroes?

Solution:

In the aforementioned polynomial, let $a^2 = x$.

Now, the polynomial becomes,

$x^2 + 4x + 5$

Comparing with $ax^2 + bx + c$,

Here, $b^2 – 4ac = 4^2 – 4(1)(5) = 16 – 20 = -4$

So, $D = b^2 – 4ac < 0$

As the discriminant (D) is negative, the given polynomial does not have real roots or zeroes.

Q.3: Compute the zeroes of the polynomial $4x^2 – 4x – 8$. Also, establish a relationship between the zeroes and coefficients.

Solution:

Let the given polynomial be $p(x) = 4x^2 – 4x – 8$

To find the zeroes, take $p(x) = 0$

Now, factorise the equation $4x^2 – 4x – 8 = 0$

$4x^2 – 4x – 8 = 0$

$4(x^2 – x – 2) = 0$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x - 2) + 1(x - 2) = 0$

$(x - 2)(x + 1) = 0$

$x = 2, x = -1$

So, the roots of $4x^2 - 4x - 8$ are -1 and 2.

Relation between the sum of zeroes and coefficients:

$-1 + 2 = 1 = -(-4)/4$ i.e. (- coefficient of x/ coefficient of x^2)

Relation between the product of zeroes and coefficients:

$(-1) \times 2 = -2 = -8/4$ i.e (constant/coefficient of x^2)

Q.4: Find the quadratic polynomial if its zeroes are 0, √5.

Solution:

A quadratic polynomial can be written using the sum and product of its zeroes as:

$x^2 - (\alpha + \beta)x + \alpha\beta$

Where α and β are the roots of the polynomial.

Here, $\alpha = 0$ and $\beta = \sqrt{5}$

So, the polynomial will be:

$x^2 - (0 + \sqrt{}5)x + 0(\sqrt{}5)$

$= x^2 - \sqrt{}5x$

Q.5: Find the value of "x" in the polynomial $2a^2 + 2xa + 5a + 10$ if $(a + x)$ is one of its factors.

Solution:

Let $f(a) = 2a^2 + 2xa + 5a + 10$

Since, $(a + x)$ is a factor of $2a^2 + 2xa + 5a + 10$, $f(-x) = 0$

So, $f(-x) = 2x^2 - 2x^2 - 5x + 10 = 0$

$-5x + 10 = 0$

$5x = 10$

$x = 10/5$

Therefore, $x = 2$

Q.6: How many zeros does the polynomial $(x - 3)^2 - 4$ have? Also, find its zeroes.

Solution:

Given polynomial is $(x - 3)^2 - 4$

Now, expand this expression.

=> $x^2 + 9 - 6x - 4$

= $x^2 - 6x + 5$

As the polynomial has a degree of 2, the number of zeroes will be 2.

Now, solve $x^2 - 6x + 5 = 0$ to get the roots.

So, $x^2 - x - 5x + 5 = 0$

=> $x(x - 1) -5(x - 1) = 0$

=> $(x - 1)(x - 5) = 0$

$x = 1, x = 5$

So, the roots are 1 and 5.

Q.7: α and β are zeroes of the quadratic polynomial $x^2 - 6x + y$. Find the value of 'y' if $3\alpha + 2\beta = 20$.

Solution:

Let, $f(x) = x^2 - 6x + y$

From the given,

$3\alpha + 2\beta = 20$————————(i)

From f(x),

$\alpha + \beta = 6$————————(ii)

And,

$\alpha\beta = y$————————(iii)

Multiply equation (ii) by 2. Then, subtract the whole equation from equation (i),

=> $\alpha = 20 - 12 = 8$

Now, substitute this value in equation (ii),

=> $\beta = 6 - 8 = -2$

Substitute the values of α and β in equation (iii) to get the value of y, such as;

$y = \alpha\beta = (8)(-2) = -16$

Q.8: If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a – b, a, a + b, then find the value of a and b.

Solution:

Let the given polynomial be:

$p(x) = x^3 - 3x^2 + x + 1$

Given,

The zeroes of the p(x) are a – b, a, and a + b.

Now, compare the given polynomial equation with general expression.

$px^3 + qx^2 + rx + s = x^3 – 3x^2 + x + 1$

Here, p = 1, q = -3, r = 1 and s = 1

For sum of zeroes:

Sum of zeroes will be = a – b + a + a + b

-q/p = 3a

Substitute the values q and p.

-(-3)/1 = 3a

a = 1

So, the zeroes are 1 – b, 1, 1 + b.

For the product of zeroes:

Product of zeroes = 1(1 – b)(1 + b)

-s/p = 1 – b^2

=> -1/1 = 1 – b^2

Or, b^2 = 1 + 1 =2

So, b = √2

Thus, 1 – √2, 1, 1 + √2 are the zeroes of equation $x^3 - 3x^2 + x + 1$.

Q.9: Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.

(i) 1/4, -1

(ii) 1, 1

(iii) 4, 1

Solution:

(i) From the formulas of sum and product of zeroes, we know,

Sum of zeroes = $\alpha + \beta$

Product of zeroes = $\alpha\beta$

Given,

Sum of zeroes = 1/4

Product of zeroes = -1

Therefore, if α and β are zeroes of any quadratic polynomial, then the polynomial can be written as:-

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (1/4)x + (-1)$

$= 4x^2 - x - 4$

Thus, $4x^2 - x - 4$ is the required quadratic polynomial.

(ii) Given,

Sum of zeroes = 1 = $\alpha + \beta$

Product of zeroes = 1 = $\alpha\beta$

Therefore, if α and β are zeroes of any quadratic polynomial, then the polynomial can be written as:-

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - x + 1$

Thus, $x^2 - x + 1$ is the quadratic polynomial.

(iii) Given,

Sum of zeroes, $\alpha + \beta = 4$

Product of zeroes, $\alpha\beta = 1$

Therefore, if α and β are zeroes of any quadratic polynomial, then the polynomial can be written as:-

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 4x + 1$

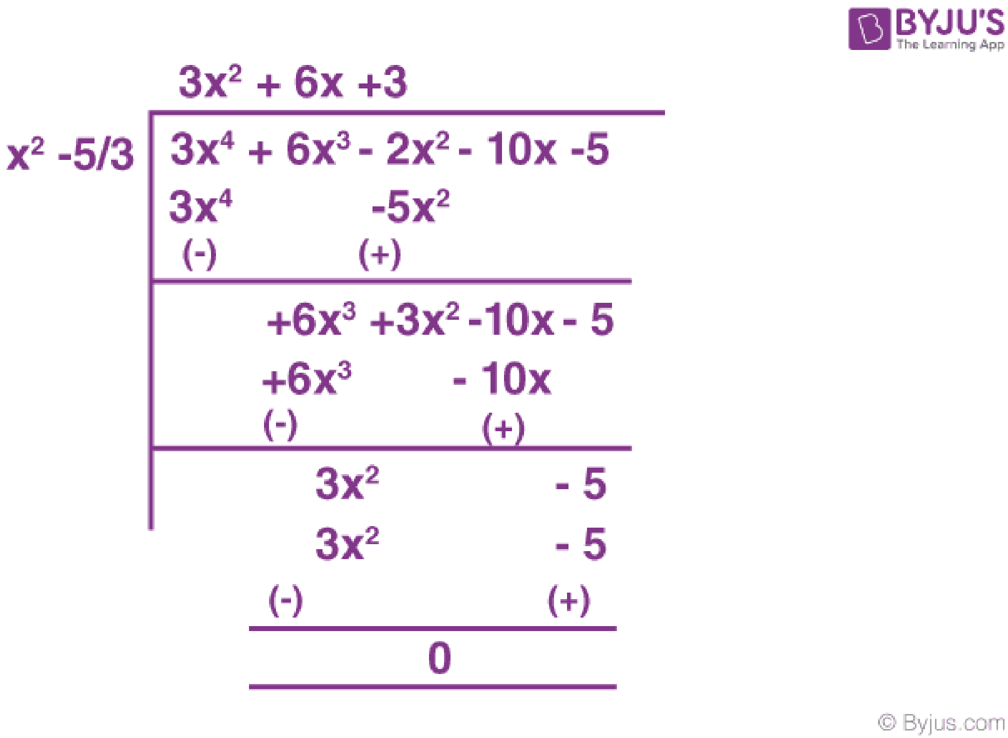
Thus, $x^2 - 4x +1$ is the quadratic polynomial.

Q.10: Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{(5/3)}$ and-$\sqrt{(5/3)}$.

Solution: Since this is a polynomial of degree 4, hence there will be a total of 4 roots.

√(5/3) and-√(5/3) are zeroes of polynomial f(x).

∴ [x-√(5/3)] [x+√(5/3)] = x^2-(5/3)



Therefore, $3x^2 + 6x + 3 = 3x(x + 1) +3 (x + 1)$

$= (3x + 3)(x + 1)$

$= 3(x + 1)(x + 1)$

$= 3(x + 1)(x + 1)$

Hence, x + 1 = 0 i.e. x = – 1 , – 1 is a zero of p(x).

So, its zeroes are given by: x = -1 and x = -1.

Therefore, all four zeroes of the given polynomial are:

√(5/3) and-√(5/3), -1 and -1.

Q.11: Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Solution:

Let α and β be the zeroes of the polynomial $f(x) = ax^2 + bx + c$.

So, $\alpha + \beta = -b/a$

$\alpha\beta = c/a$

According to the given, $1/\alpha$ and $1/\beta$ are the zeroes of the required quadratic polynomial.

Now, the sum of zeroes = $(1/\alpha) + (1/\beta)$

$= (\alpha + \beta)/\alpha\beta$

$= (-b/a)/ (c/a)$

$= -b/c$

Product of two zeroes = $(1/\alpha)\ (1/\beta)$

$= 1/\alpha\beta$

$= 1/(c/a)$

$= a/c$

The required quadratic polynomial = $k[x^2 – (\text{sum of zeroes})x + (\text{product of zeroes})]$

$= k[x^2 - (-b/c)x + (a/c)]$

$= k[x^2 + (b/c) + (a/c)]$

Q.12: Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Solution:

Given,

$f(x) = 3x^2 - x^3 - 3x + 5$

$g(x) = x - 1 - x^2$

Dividing $f(x) = 3x^2 - x^3 - 3x + 5$ by $g(x) = x - 1 - x^2$

$$
\begin{array}{r|rrrr}
 & x & -2 & & \\
\hline
-x^2 + x - 1 & -x^3 & +3x^2 & -3x & +5 \\
 & - & & & \\
 & -x^3 & +x^2 & -x & \\
\hline
 & & 2x^2 & -2x & +5 \\
 & & - & & \\
 & & 2x^2 & -2x & +2 \\
\hline
 & & & & 3
\end{array}
$$

Here,

Quotient $= q(x) = x - 2$

Remainder $= r(x) = 3$

By division algorithm of polynomials,

Dividend = (Quotient × Divisor) + Remainder

So,

$[q(x) \times g(x)] + r(x) = (x - 2)(x - 1 - x^2) + 3$

$= x^2 - x - x^3 - 2x + 2 + 2x^2 + 3$

$= 3x^2 - x^3 - 3x + 5$

$= f(x)$

Hence, the division algorithm is verified.

Q.13: For what value of k, is the polynomial $f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$ completely divisible by $3x^2 - 5$?

Solution:

Given,

$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k$

$g(x) = 3x^2 - 5$

Dividing f(x) by g(x),

$$
\begin{array}{rrrrrrr}
 & & x^2 & -3x & +2 & & \\
\cline{3-7}
3x^2-5 & \big) & 3x^4 & -9x^3 & +x^2 & +15x & +k \\
 & - & & & & & \\
 & & 3x^4 & +0x^3 & -5x^2 & & \\
\hline
 & & & -9x^3 & +6x^2 & +15x & +k \\
 & & - & & & & \\
 & & & -9x^3 & +0x^2 & +15x & \\
\hline
 & & & & 6x^2 & +0x & +k \\
 & & & - & & & \\
 & & & & 6x^2 & +0x & -10 \\
\hline
 & & & & & & k+10
\end{array}
$$

Given that f(x) is completely divisible by $3x^2 - 5$.

So, the remainder = 0

k + 10 = 0

k = -10

Q.14: If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Solution:

Given cubic polynomial is $p(x) = x^3 - 3x^2 - 10x + 24$

4 is a zero of p(x).

So, (x – 4) is the factor of p(x).

Let us divide the given polynomial by (x – 4).

$$\begin{array}{r|rrrr} & x^2 & +x & -6 & \\ \hline x-4 & x^3 & -3x^2 & -10x & +24 \\ - & & & & \\ & x^3 & -4x^2 & & \\ \hline & & x^2 & -10x & +24 \\ & - & & & \\ & & x^2 & -4x & \\ \hline & & & -6x & +24 \\ & & - & & \\ & & & -6x & +24 \\ \hline & & & & 0 \end{array}$$

Here, the quotient = $x^2 + x - 6$

= $x^2 + 3x - 2x - 6$

= x(x + 3) – 2(x + 3)

= (x – 2)(x + 3)

Therefore, the other two zeroes of the given cubic polynomial are 2 and -3.

Important 2 Marks Questions for Class 10 Maths Board are as follows-

Q.1: Find the value of k for which the roots of the quadratic equation

, will have equal value.

Solution:

Given,

$2x^2 + kx + 8 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = 2, b = k, c = 8$

Condition for the equal roots is:

$b^2 - 4ac = 0$

$k^2 - 4(2)(8) = 0$

$k^2 - 64 = 0$

$k^2 = 64$

$k = \pm 8$

Q.2: Determine the AP whose third term is 5 and the seventh term is 9.

Solution:

Let a be the first term and d be the common difference of an AP.

Given,

Third term = 5

$a + 2d = 5$….(i)

Seventh term = 9

$a + 6d = 9$….(ii)

Subtracting (i) from (ii),

$a + 6d - a - 2d = 9 - 5$

$4d = 4$

$d = 1$

Substituting $d = 1$ in (i),

$a + 2(1) = 5$

$a = 5 - 2 = 3$

Therefore, the AP is: 3, 4, 5, 6,…

Q.3: Find a relation between x and y if the points A(x, y), B(−4, 6) and C(−2, 3) are collinear.

Solution:

Let the given points be:

$A(x, y) = (x_1, y_1)$

$B(-4, 6) = (x_2, y_2)$

$C(-2, 3) = (x_3, y_3)$

If three points are collinear then the area of the triangle formed by these points is 0.

i.e. $(½)\ |x_1(y_2 – y_3) + x_2(y_3 – y_1) + x_3(y_1 – y_2)| = 0$

$(½)\ |x(6 – 3) + (-4)(3 – y) + (-2)(y – 6)| = 0$

$x(3) – 4(3 – y) – 2(y – 6) = 0$

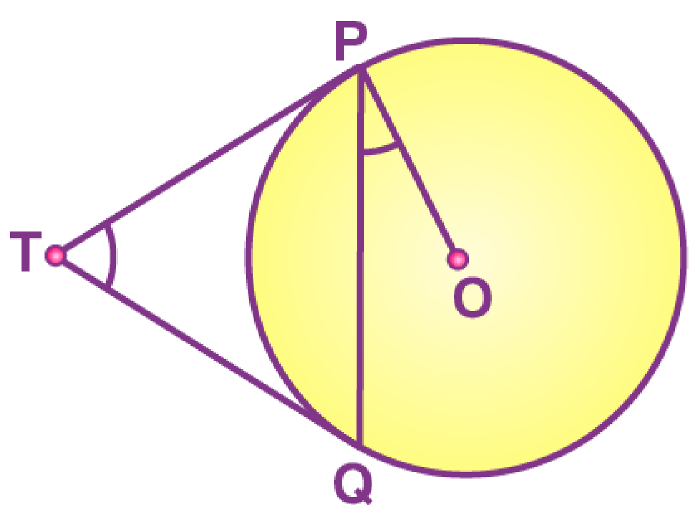
$3x – 12 + 4y – 2y + 12 = 0$

$3x + 2y = 0$

Or

$x = -2y/3$

Q.4: In the figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T, prove that ∠PTQ = 2OPQ.



Solution:

Given that two tangents TP and TQ are drawn to a circle with centre O from an external point T

Let $\angle PTQ = \theta$.

Now, by using the theorem "the lengths of tangents drawn from an external point to a circle are equal", we can say TP = TQ. So, TPQ is an isosceles triangle.

Thus,

$\angle TPQ = \angle TQP = ½ (180° - \theta) = 90° – (½) \theta$

By using the theorem, "the tangent at any point of a circle is perpendicular to the radius through the point of contact", we can say $\angle OPT = 90°$

Therefore,

$\angle OPQ = \angle OPT – \angle TPQ = 90° – [90° – (½) \theta]$

$\angle OPQ = (½)\theta$

$\angle OPQ = (½) \angle PTQ$

$\Rightarrow \angle PTQ = 2 \angle OPQ$.

Hence proved.

Q.5: A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. [Use π = 22/7]

Solution:

Length of the arc = Length of wire

$(\theta/360°)\ 2\pi r = 22$

$(60°/360°) \times 2 \times (22/7) \times r = 22$

$(1/6) \times (2/7) \times r = 1$

$r = (7/2) \times 6$

$r = 21$

Therefore, the radius of the circle is 21 cm.

Q.6: If a number x is chosen at random from the numbers −3, −2, −1, 0, 1, 2, 3, what is the probability that x2 ≤ 4?

Solution:

Sample space = S = {−3, −2, −1, 0, 1, 2, 3}

n(S) = 7

Let E be the event of choosing a number x such that x2 ≤ 4.

i.e. x ≤ ±2

E = {−2, −1, 0, 1, 2}

n(E) = 5

P(E) = n(E)/n(S) = 5/7

Hence, the required probability is 5/7.

Q.7: The larger of two supplementary angles exceeds the smaller by 18°. Find the angles.

Solution:

Let x and (x + 18°) be the supplementary angles.

That means,

x + (x + 18°) = 180°

2x = 180° – 18°

2x = 162°

x = 162°/2

x = 81°

Now, x + 18° = 81° + 18° = 99°

Therefore, the supplementary angles are 81° and 99°.

Q.8: Find the mean of the following distribution:

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	5	10	10	7	8

Solution:

Class	Frequency (f_i)	Midpoint (x_i)	$f_i x_i$
3 – 5	5	4	20
5 – 7	10	6	60
7 – 9	10	8	80
9 – 11	7	10	70
11 – 13	8	12	96
Total	$\sum f_i = 40$	$\sum x_i = 40$	$\sum f_i x_i = 326$

Mean = $\sum f_i x_i / \sum f_i$

= 326/40

= 8.15

Therefore, the mean of the given distribution is 8.15.

Q.9: Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

Solution:

Let $(5 + 3\sqrt{2})$ be a rational number.

$5 + 3\sqrt{2} = p/q$ (Where $q \neq 0$ and p and q are co- prime numbers)

$3\sqrt{2} = (p/q) - 5$

$3\sqrt{2} = (p - 5q)/q$

$\sqrt{2} = (p - 5q)/3q$

p and q are integers and $q \neq 0$

Thus, $(p - 5q)/3q$ is rational number.

Also, $\sqrt{2}$ is a rational number.

However, it is given that $\sqrt{2}$ is an irrational number.

This is a contradiction, and thus, our assumption that $(5 + 3\sqrt{2})$ be a rational number is wrong.

That means $(5+3\sqrt{2})$ is an irrational number.

Hence proved.

Q.10: Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

Solution:

Given quadratic equation is:

$px^2 - 14x + 8 = 0$

Let α and 6α be the roots of the given quadratic equation.

Sum of the roots = -coefficient of x/coefficient of x^2

$\alpha + 6\alpha = -(-14)/p$

$7\alpha = 14/p$

$\alpha = 2/p$....(i)

Product of roots = constant term/coefficient of x^2

$(\alpha)(6\alpha) = 8/p$

$6\alpha^2 = 8/p$

Substituting $\alpha = 2/p$ from (i),

$6 \times (2/p)^2 = 8/p$

$24/p^2 = 8/p$

$3/p = 1$

$p = 3$

Therefore, the value of p is 3.

Important 4 Marks Questions and Solutions for Class 12 Maths Board are as follows-

Question 1- If mth term of an A.P. is

and nth term is

, then find the sum of its first mn terms.

Solution:

Let a be the first term and d be the common difference of an AP.

Given,

$a_m = 1/n$

i.e. $a + (m - 1)d = 1/n$

$n[a + (m - 1)d] = 1$

$an + mnd - nd = 1$….(i)

And

$a_n = 1/m$

$a + (n - 1)d = 1/m$

$m[a + (n - 1)d] = 1$

$am + mnd - md = 1$….(ii)

From (i) and (ii),

$an + mnd - nd = am + mnd - md$

$an - am = nd - md$

$a(n - m) = (n - m)d$

$a = d$

Substituting $a = d$ in equation (i),

$dn + mnd - nd = 1$

$mnd = 1$

$d = 1/mn$

Therefore, $a = 1/mn$

Sum of first mn terms is:

$S_{mn} = (mn/2)[2(1/mn) + (mn - 1)(1/mn)]$

$= (mn/2)[(2/mn) + (mn - 1)/mn]$

$= (mn/2mn)[2 + mn - 1]$

$= (1/2)(mn + 1)$

Hence, the sum of first mn terms = $(mn +)/2$.

Question 2- A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from

to

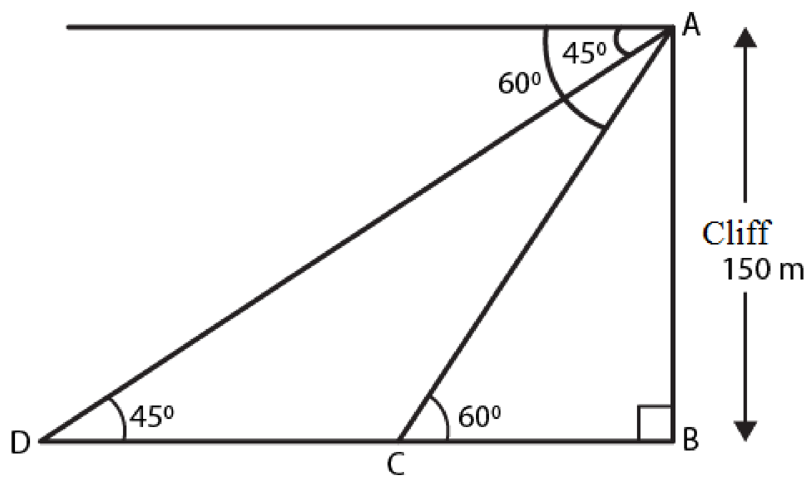
in 2 minutes. Find the speed of the boat in m/h.

Solution:

Let AB be the height of the cliff.

AB = 150 m

C and D be the position of a boat.



Boat is moved from C to D in 2 minutes = 2/60 hr = 1/30 hr

In right triangle ABC,

tan 60° = AB/BC

√3 = 150/BC

BC = 150/√3 m

In right triangle ABD,

tan 45° = AB/AD

1 = 150/(BC + CD)

(150/√3) + CD = 150

CD = 150 – (150/√3)

CD = (150√3 – 150)/√3

= 150(√3 – 1)/√3 m

Distance CD = Speed × time

150(√3 – 1)/√3 = Speed × (1/30)

Speed = 30 × [150(√3 – 1)/√3]

= 1500√3(√3 – 1) m/h

Question 3- Two different dice are thrown together. Find the probability that the numbers obtained

(i) have a sum less than 7

(ii) have a product less than 16

(iii) is a doublet of odd numbers.

Solution:

Given,

Two different dice are thrown together.

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total number of outcomes = n(S) = 36

(i) have a sum less than 7

Let E_1 be the event of getting the numbers on dice whose sum is less than 7.

E_1 = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2),(5, 1)}

$n(E_1) = 15$

$P(E_1) = n(E_1)/n(S) = 15/36 = 5/12$

(ii) have a product less than 16

Let E_2 be the event of getting the numbers on dice whose product is less than 16.

E_2 = {1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2)}

$n(E_2) = 25$

$P(E_2) = n(E_2)/n(S) = 25/36$

(iii) is a doublet of odd numbers.

Let E_3 be the event of getting a doublet of odd numbers.

E_3 = {(1, 1), (3, 3), (5, 5)}

$P(E_3) = n(E_3)/n(S) = 3/36 = 1/12$

Question 4- The area of a triangle is 5 sq units. Two of its vertices are (2,1) and (3,-2). If the third vertex is

, then find the value of y.

Solution:

Let A, B, and C be the vertices of a triangle.

$A(2, 1) = (x_1, y_1)$

$B(3, -2) = (x_2, y_2)$

$C(7/2, y) = (x_3, y_3)$

Given,

Area of the triangle is 5 sq.units.

$(1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 5$

$(1/2)|2(-2 - y) + 3(y - 1) + (7/2)(1 + 2)| = 5$

$|-4 - 2y + 3y - 3 + (7/2)(3)| = 10$

$y - 7 + (21/2) = 10$

$y = 10 + 7 - (21/2)$

$y = 17 - (21/2)$

$y = (34 - 21)/2$

$y = 13/2$

Therefore, the value of y is 13/2.

Question 5- The

th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with an internal radius of 10 cm. Find the height of water in the cylindrical vessel.

Solution:

Given,

Height of conical vessel = h = 24 cm

Radius of conical vessel = r = 5 cm

Volume of water = (3/4) × Volume of conical vessel

$= (3/4) \times (1/3)\pi r^2 h$

$= (3/4) \times (1/3) \times (\pi \times 5 \times 5 \times 24)$

$= 150\pi\ cm^3$

Let H be the height of a cylindrical vessel.

Radius of cylindrical vessel = R = 10 cm {given}

From the given,

Volume of cylindrical vessel = Volume of water

$\pi R^2 H = 150\pi$

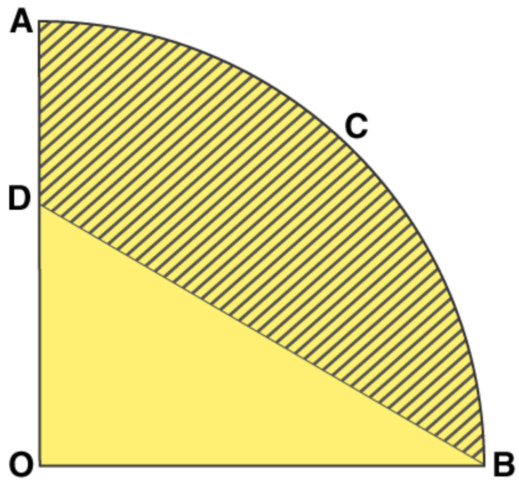
$R^2 H = 150$

$10 \times 10 \times H = 150$

H = 1.5 cm

Therefore, the height of a cylindrical vessel.

Question 6- In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the shaded region.



Solution:

Given,

Radius of the quadrant (r) = 3.5 cm

Area of quadrant = $(1/4)\pi r^2$

= (1/4) × (22/7) × 3.5 × 3.5

= 9.625 cm^2

Area of triangle BOD = (1/2) × OB × OD

= (1/2) × 3.5 × 2

= 3.5 cm^2

Area of the shaded region = Area of quadrant – Area of triangle BOD

= 9.625 – 3.5

$= 6.125\ cm^2$

Question 7- If the equation

has equal roots then show that

.

Solution:

Given,

$(1 + m^2)x^2 + 2mcx + c^2 – a^2 = 0$

Comparing with the standard form of a quadratic equation $ax^2 + bx + c = 0$,

$a = (1 + m^2)$

$b = 2mc$

$c = (c^2 – a^2)$

Given that the quadratic equation has equal roots.

So, $b^2 – 4ac = 0$ {determinant is equal to 0}

$(2mc)^2 – 4(1 + m^2)(c^2 – a^2) = 0$

$4m^2c^2 – 4(c^2 – a^2 + m^2c^2 – m^2a^2) = 0$

$4m^2c^2 – 4c^2 + 4a^2 – 4m^2c^2 + 4m^2a^2 = 0$

$4c^2 = 4a^2m^2 + 4a^2$

$4c^2 = 4a^2(m^2 + 1)$

$c^2 = a^2(1 + m^2)$

Question 8- Find the sum of n terms of the series

Solution:

Given series:

[4 – (1/n)] + [4 – (2/n)] + [4 – (3/n)] + ... up to n terms

= (4 + 4 + 4 + ... up to n terms) – [(1/n) + (2/n) + (3/n) + ... up to n terms]

= 4n – (1/n)[1 + 2 + 3 + ... up to n terms]

= 4n – (1/n)[n(n + 1)/2]   {sum of first n natural numbers = n(n + 1)/2}

= 4n – (n + 1)/2

= (8n – n – 1)/2

= (7n – 1)/2

Therefore, the sum of the given series is (7n – 1)/2.

Question 9- Prove that: (sec A – cos A). (cot A + tan A) = tan A. sec A

Solution:

LHS = (sec A – cos A)(cot A + tan A)

= [(1/cos A) – cos A] [(cos A/sin A) + (sin A/cos A)]

$= [(1 - \cos^2 A)/\cos A]\ [(\cos^2 A + \sin^2 A)/ (\sin A \cos A)]$

$= (\sin^2 A/ \cos A)\ [1/(\sin A \cos A)]$

$= \sin A/(\cos A \cos A)$

$= (\sin A/\cos A)\ (1/\cos A)$

$= \tan A \sec A$

= RHS

Thus, (sec A – cos A). (cot A + tan A) = tan A. sec A

Hence proved.

Question 10- The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f.

Daily pocket allowance (in Rs.)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of children	7	6	9	13	f	5	4

Solution:

Class interval	Number of children (f_i)	Mid-point (x_i)	$f_i x_i$
11 – 13	7	12	84

13 – 15	6	14	84
15 – 17	9	16	144
17 – 19	13	18	234
19 – 21	f	20	20f
21 – 23	5	22	110
23 – 25	4	24	96
Total	$\Sigma f_i = 44 + f$		$\Sigma f_i x_i = 752 + 20f$

Mean = $\bar{x} = \Sigma f_i x_i / \Sigma f_i$

= (752 + 20f)/(44 + f)

Given that, (752 + 20f)/(44 + f) = 18

752 + 20f = 18(44 + f)

752 + 20f = 792 + 18f

20f – 18f = 792 – 752

2f = 40

f = 20

Therefore, the missing frequency = f = 20

Important 4 Marks Questions and Solutions for Class 10 Maths Board are as follows-

Question 1- Solve for x:

Solution:

Taking LCM and cross multiplying the terms,

$x[x - 1 + 2x - 4] = 6[x^2 - 2x - x + 2]$

$x(3x - 5) = 6(x^2 - 3x + 2)$

$6x^2 - 18x + 12 - 3x^2 + 5x = 0$

$3x^2 - 13x + 12 = 0$

$3x^2 - 9x - 4x + 12 = 0$

$3x(x - 3) - 4(x - 3) = 0$

$(3x - 4)(x - 3) = 0$

$3x - 4 = 0, x - 3 = 0$

$x = 4/3, 3$

Question 2-

Find the sum of first 24 terms of an A.P. whose nth term is given by

.

Solution:

Given,

$a_n = 3 + 2n$

Substituting n = 1, 2, 3,…

When n = 1,

$a_1 = 3 + 2(1) = 3 + 2 = 5$

When n = 2,

$a_2 = 3 + 2(2) = 3 + 4 = 7$

When n = 3,

$a_3 = 3 + 2(3) = 3 + 6 = 9$

Thus, the AP is: 5, 7, 9,…

Here,

First term = a = 5

Common difference = d = 2

We know that,

Sum of first n terms of an AP = $S_n = (n/2) [2a + (n - 1)d]$

Now,

S_{24} = (24/2) [2(5) + (24 – 1)(2)]

= 12[10 + 23(2)]

= 12[10 + 46]

= 12 × 56

= 672

Therefore, the sum of the first 24 terms of the given AP is 672.

Question 3-

A bucket, is in the form of a frustum of a cone whose height is 42 cm and the radii of its circular ends are 30 cm and 10 cm. Find the amount of milk (in litres) which this bucket can hold. If the milkman sells the milk at the rate of Rs. 40 per litre, what amount he will get from the sale?

If the milkman sells half the milk at less rate to the economically weaker section of society, what value he exhibits by doing this?

Solution:

Given that a bucket is in the form of a frustum of a cone with the following dimensions.

Height = h = 42 cm

R = 30 cm

r = 10 cm

Volume of frustum of a cone = (1/3)πh[R^2 + r^2 + Rr]

Substituting the values,

= (1/3) × (22/7) × 42 × [$(30)^2$ + $(10)^2$ + (30)(10)]

= 44 × [900 + 100 + 300]

= 44 × 1300

= 572000 cm^3

= (572000/1000) litres {since 1 cm^3 = 1/1000 litres}

= 57.2 litres

Therefore, the bucket can hold 57.2 litres of milk.

Also,

Cost of 1 litre milk = Rs. 40 (given)

So, the cost of 57.2 litres of milk = 57.2 × Rs. 40 = Rs. 2288

Hence, the milkman can get Rs. 2288 from selling the milk in the bucket.

If the milkman sells half the milk at less rate to the economically weaker section of society, he shows the values of help, kindness, and empathy.

Question 4-

If the coordinates of two points are A(3,4), B (5,-2) and a point P (x,5) is such that PA = PB, then find the area of

.

Solution:

Given points:

A(3, 4), B(5, -2) and P(x, 5)

Also,

PA = PB

Squaring on both sides,

$PA^2 = PB^2$

Using distance formula, $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$;

$(x - 3)^2 + (5 - 4)^2 = (x - 5)^2 + (5 + 2)^2$

$x^2 + 9 - 6x + 1 = x^2 + 25 - 10x + 49$

$10 - 6x = 74 - 10x$

$\Rightarrow 10x - 6x = 74 - 10$

$\Rightarrow 4x = 64$

$\Rightarrow x = 16$

$\Rightarrow P = (x, 5) = (16, 5)$

Let,

$P(16, 5) = (x_1, y_1)$, $A(3, 4) = (x_2, y_2)$, $B(5, -2) = (x_3, y_3)$

Area of triangle PAB = $(1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= (1/2)|16(4 + 2) + 3(-2 - 5) + 5(5 - 4)|$

$= (1/2)|16(6) + 3(-7) + 5(1)|$

$= (1/2)|96 - 21 + 5|$

$= (1/2) \times 80$

= 40 square units

Question 5-

A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys so formed.

Solution:

Given,

Height of cylinder = H = 15 cm

Diameter = 12 cm

Radius of cylinder = R = 12/2 = 6 cm

Radius of cone = 3 cm

Height of cone = h = 9 cm

Let n be the number of toys in the shape of a cone made.

So,

Volume of cylinder = n × volume of a cone

$\pi R^2 H = (1/3)\pi r^2 h$

6 × 6 × 15 = n × (1/3) × 3 × 3 × 9

n = (6 × 6 × 15 × 3)/(3 × 3 × 9)

n = 20

Question 6-

From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed. The remaining cards are mixed and a card is drawn at random. Find the probability that the drawn card is

(i) a black Queen

(ii) a card of red colour

(iii) a Jack of black colour

(iv) a face card

Solution:

We know that, P(an event) = Number of favourable outcomes/Total number of outcomes

Given,

From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed.

So, the total number of outcomes = 52 – (2 + 2 + 2 + 2) = 52 – 8 = 44

(i) a black Queen

Number of black queen cards in the available cards = 0

P(getting a black queen) = 0/44 = 0

(ii) a card of red colour

Number of red colour cards = 26 – 4 = 22

P(getting a card of red colour) = 22/44 = 1/2

(iii) a Jack of black colour

Number of black Jack cards = 2

P(getting a Jack of black colour) = 2/44 = 1/22

(iv) a face card

Number of face cards = 12 – 6 = 6

P(getting a face card) = 6/44 = 3/22

Question 7-

From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some vertically over the top the building is

and

respectively. Find the height of the helicopter above the ground.

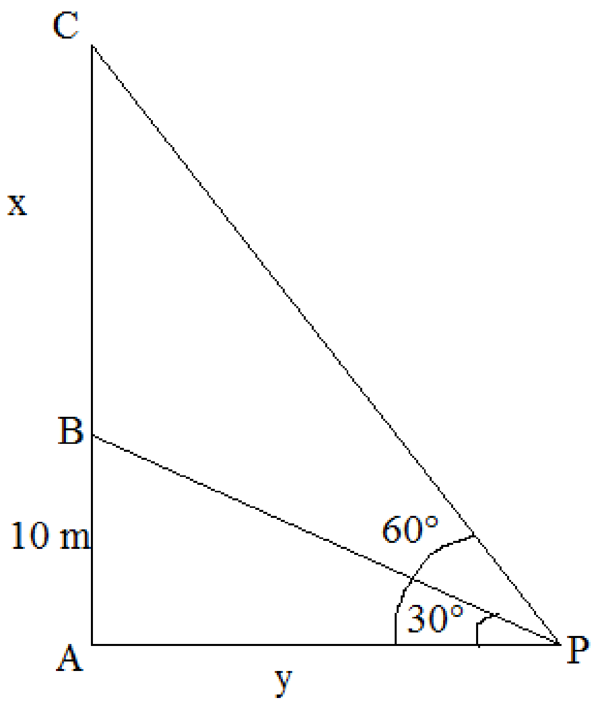
Solution:

Let P be the position of a point and C be the position of a helicopter.

Height of the building AB = 10 m

AC = x m

PA = y m



In right triangle PAB,

tan 30° = AB/PA

$1/\sqrt{3} = 10/y$

$y = 10\sqrt{3}$ m

In right triangle PAC,

tan 60° = AC/PA

$\sqrt{3} = (x + 10)/10\sqrt{3}$

$10\sqrt{3}\,\sqrt{3} = x + 10$

$x = 30 - 10 = 20$

Therefore, the height of helicopter from the ground = x + 10 = 20 + 10 = 30 m

Question 8-

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Solution:

Steps of Construction:

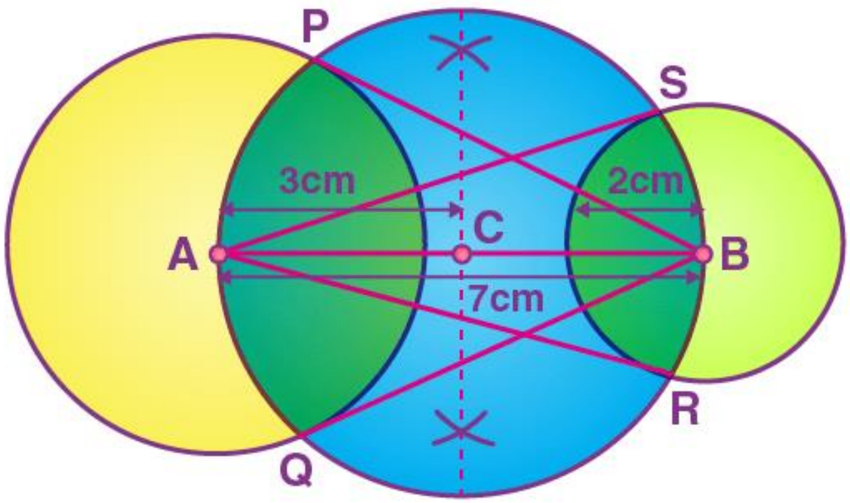
Step 1: Draw a line segment AB of 7 cm.

Step 2: Taking A and B as centres, draw two circles of 3 cm and 2 cm radius respectively.

Step 3: Bisect the line AB. Let the midpoint of AB be C.

Step 4: Taking C as centre, draw a circle of radius AC which intersects the two circles at point P,Q,R and S.

Step 5: Join BP, BQ, AS and AR.



Here, BP, BQ, AS and AR are the tangents to the circles.

Question 9-

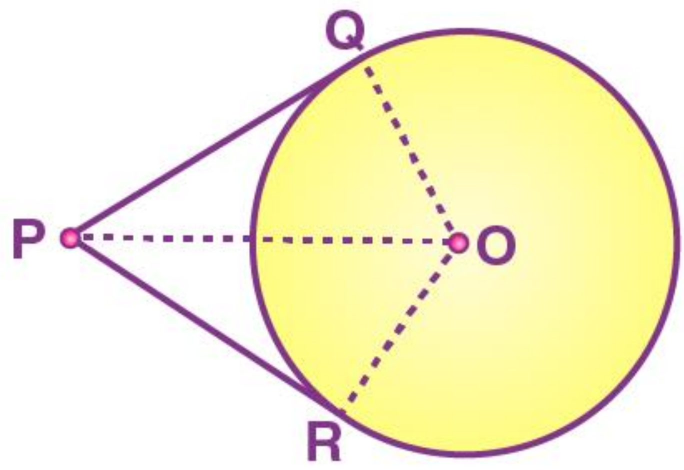
Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:

Consider a circle with the centre "O" and P is the point that lies outside the circle. Hence, the two tangents formed are PQ and PR.

We need to prove: PQ = PR.

To prove the tangent PQ is equal to PR, join OP, OQ and OR. Hence, ∠OQP and ∠ORP are the right angles.



Therefore, OQ = OR (Radii)

OP = OP (Common side)

By using the RHS rule, we can say, Δ OQP ≅ Δ ORP.

Thus, by using the CPCT rule, the tangent PQ = PR.

Hence proved.

Question 10-

The Life insurance agent found the following data for the distribution of ages of 100 policyholders. Calculate the median age, if policies are given only to the persons whose age is 18 years onwards but less than the 60 years.

Age (in years)	Number of policyholders

Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Solution:

Let us calculate the class intervals and the corresponding frequencies for the given data.

Class interval	Frequency	Cumulative frequency
15 – 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45

35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100

N = 100

N/2 = 100/2 = 50

The cumulative frequency greater than and nearer to 50 is 78 which belongs to the class interval 35 – 40.

So, the median class is 35 – 40.

Frequency of the median class = f = 33

Lower limit of median class = l = 35

Cumulative frequency of the class preceding the median class = cf = 45

Class size = h = 5

Median = l + [(N/2 – cf)/f] × h

= 35 + [(50 – 45)/33] × 5

= 35 + (25/33)

= 35 + 0.75

= 35.75 (approx.)

Therefore, the median age = 35.75 years.