

# Class 10 Math Chapter 3 Important Questions With

## Answers

**Q.1: The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs.160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs.300. Represent the situation algebraically.**

**Solution:**

Let the cost of 1 kg of apples be 'Rs. x'.

And, let the cost of 1 kg of grapes be 'Rs. y'.

According to the question, the algebraic representation is

$$2x + y = 160$$

$$\text{And } 4x + 2y = 300$$

For,  $2x + y = 160$  or  $y = 160 - 2x$ , the solution table is;

x	50	60	70
y	60	40	20

For  $4x + 2y = 300$  or  $y = (300 - 4x) / 2$ , the solution table is;

x	70	80	75
y	10	-10	0

Note: Students can also represent these two equations graphically, by using the given points of x-coordinate and y-coordinate.

**Q.2: Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.**

**Solution:**

Given, half the perimeter of a rectangular garden = 36 m

$$\text{so, } 2(l + b)/2 = 36$$

$$(l + b) = 36 \dots\dots\dots(1)$$

Given, the length is 4 m more than its width.

Let width = x

And length = x + 4

Substituting this in eq(1), we get;

$$x + x + 4 = 36$$

$$2x + 4 = 36$$

$$2x = 32$$

$$x = 16$$

Therefore, the width is 16 m and the length is  $16 + 4 = 20$  m.

**Q.3: On comparing the ratios  $a_1/a_2$ ,  $b_1/b_2$ , and  $c_1/c_2$ , find out whether the following pair of linear equations are consistent, or inconsistent.**

**(i)  $3x + 2y = 5$  ;  $2x - 3y = 7$**

**(ii)  $2x - 3y = 8$  ;  $4x - 6y = 9$**

**Solution:**

(i) Given :  $3x + 2y = 5$  or  $3x + 2y - 5 = 0$

and  $2x - 3y = 7$  or  $2x - 3y - 7 = 0$

Comparing the above equations with  $a_1x + b_1y + c_1 = 0$

And  $a_2x + b_2y + c_2 = 0$

We get,

$a_1 = 3, b_1 = 2, c_1 = -5$

$a_2 = 2, b_2 = -3, c_2 = -7$

$a_1/a_2 = 3/2, b_1/b_2 = 2/-3, c_1/c_2 = -5/-7 = 5/7$

Since,  $a_1/a_2 \neq b_1/b_2$  the lines intersect each other at a point and have only one possible solution.

Hence, the equations are consistent.

(ii) Given  $2x - 3y = 8$  and  $4x - 6y = 9$

Therefore,

$a_1 = 2, b_1 = -3, c_1 = -8$

$a_2 = 4, b_2 = -6, c_2 = -9$

$a_1/a_2 = 2/4 = 1/2, b_1/b_2 = -3/-6 = 1/2, c_1/c_2 = -8/-9 = 8/9$

Since,  $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Therefore, the lines are parallel to each other and they have no possible solution. Hence, the equations are inconsistent.

**Q.4: Solve the following pair of linear equations by the substitution method.**

**(i)  $x + y = 14$**

**$x - y = 4$**

**(ii)  $3x - y = 3$**

**$9x - 3y = 9$**

**Solution:**

(i) Given,

$x + y = 14$  and  $x - y = 4$  are the two equations.

From 1st equation, we get,

$$x = 14 - y$$

Now, put the value of  $x$  in second equation to get,

$$(14 - y) - y = 4$$

$$14 - 2y = 4$$

$$2y = 10$$

$$\text{Or } y = 5$$

By the value of  $y$ , we can now find the value of  $x$ ;

$$\therefore x = 14 - y$$

$$\therefore x = 14 - 5$$

$$\text{Or } x = 9$$

Hence,  $x = 9$  and  $y = 5$ .

(ii) Given,

$3x - y = 3$  and  $9x - 3y = 9$  are the two equations.

From 1st equation, we get,

$$x = (3 + y)/3$$

Now, substitute the value of  $x$  in the given second equation to get,

$$9[(3 + y)/3] - 3y = 9$$

$$\Rightarrow 3(3+y) - 3y = 9$$

$$\Rightarrow 9 + 3y - 3y = 9$$

$$\Rightarrow 9 = 9$$

Therefore,  $y$  has infinite values and since,  $x = (3 + y)/3$ , so  $x$  also has infinite values.

**Q.5: Solve  $2x + 3y = 11$  and  $2x - 4y = -24$  and hence find the value of 'm' for which  $y = mx + 3$ .**

**Solution:**

$$2x + 3y = 11 \dots\dots\dots (i)$$

$$2x - 4y = -24 \dots\dots\dots (ii)$$

From equation (i), we get;

$$x = (11 - 3y)/2 \dots\dots\dots (iii)$$

Putting the value of  $x$  in equation (ii), we get

$$2[(11 - 3y)/2] - 4y = -24$$

$$11 - 3y - 4y = -24$$

$$-7y = -35$$

$$y = 5 \dots\dots\dots (iv)$$

Putting the value of y in equation (iii), we get;

$$x = (11 - 15)/2 = -4/2 = -2$$

$$\text{Hence, } x = -2, y = 5$$

Also,

$$y = mx + 3$$

$$5 = -2m + 3$$

$$-2m = 2$$

$$m = -1$$

Therefore, the value of m is -1.

**Q.6: The coach of a cricket team buys 7 bats and 6 balls for Rs.3800. Later, she buys 3 bats and 5 balls for Rs.1750. Find the cost of each bat and each ball.**

**Solution:**

Let the cost of a bat be x and the cost of a ball be y.

According to the question,

$$7x + 6y = 3800 \dots\dots\dots (i)$$

$$3x + 5y = 1750 \dots\dots\dots (ii)$$

From (i), we get;

$$y = (3800 - 7x)/6 \dots\dots\dots (iii)$$

Substituting (iii) in (ii). we get,

$$3x + 5[(3800 - 7x)/6] = 1750$$

$$\Rightarrow 3x + (9500/3) - (35x/6) = 1750$$

$$3x - (35x/6) = 1750 - (9500/3)$$

$$(18x - 35x)/6 = (5250 - 9500)/3$$

$$\Rightarrow -17x/6 = -4250/3$$

$$\Rightarrow -17x = -8500$$

$$x = 500$$

Putting the value of x in (iii), we get;

$$y = (3800 - 7 \times 500)/6 = 300/6 = 50$$

Hence, the cost of a bat is Rs 500 and the cost of a ball is Rs 50.

**Q.7: A fraction becomes  $9/11$  if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes  $5/6$ . Find the fraction.**

**Solution:**

Let the fraction be  $x/y$ .

According to the question,

$$(x + 2)/(y + 2) = 9/11$$

$$11x + 22 = 9y + 18$$

$$11x - 9y = -4 \dots\dots\dots (1)$$

$$(x + 3)/(y + 3) = 5/6$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = -3 \dots\dots\dots (2)$$

From (1), we get

$$x = (-4 + 9y)/11 \dots\dots\dots (3)$$

Substituting the value of x in (2), we get

$$6[(-4 + 9y)/11] - 5y = -3$$

$$-24 + 54y - 55y = -33$$

$$-y = -9$$

$$y = 9 \dots\dots\dots (4)$$

Substituting the value of y in (3), we get

$$x = (-4 + 81)/11 = 77/11 = 7$$

Hence, the fraction is 7/9.

**Q.8 Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:**

**(i) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?**

**Solution:**

Let us assume, the present age of Nuri be x.

And the present age of Sonu is y.

According to the given condition, we can write as;

$$x - 5 = 3(y - 5)$$



$$x - 3y = -10 \dots\dots\dots(1)$$

Now,

$$x + 10 = 2(y + 10)$$

$$x - 2y = 10 \dots\dots\dots(2)$$

Subtract eq. 1 from 2, to get,

$$y = 20 \dots\dots\dots(3)$$

Substituting the value of y in eq.1, we get,

$$x - 3(20) = -10$$

$$x - 60 = -10$$

$$x = 50$$

Therefore,

The age of Nuri is 50 years

The age of Sonu is 20 years.

**(ii) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs.27 for a book kept for seven days, while Susy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.**

**Solution:**

Let the fixed charge for the first three days be Rs. A and the charge for each day extra be Rs. B.

According to the information given,

$$A + 4B = 27 \dots\dots\dots (i)$$

$$A + 2B = 21 \dots\dots\dots (ii)$$

When equation (ii) is subtracted from equation (i) we get,

$$2B = 6$$

$$B = 3 \text{ .....(iii)}$$

Substituting  $B = 3$  in equation (i) we get,

$$A + 12 = 27$$

$$A = 15$$

Hence, the fixed charge is Rs. 15.

And the Additional charge per day is Rs. 3.

**Q.9: Solve the following pair of linear equations by the substitution and cross-multiplication methods:**

$$8x + 5y = 9$$

$$3x + 2y = 4$$

**Solution:**

$$8x + 5y = 9 \text{ .....(1)}$$

$$3x + 2y = 4 \text{ .....(2)}$$

From equation (2) we get;

$$x = (4 - 2y) / 3 \text{ ..... (3)}$$

Substituting this value in equation 1, we get

$$8[(4 - 2y)/3] + 5y = 9$$

$$32 - 16y + 15y = 27$$

$$-y = -5$$

$$y = 5 \text{ .....(4)}$$

Substituting this value in equation (2), we get

$$3x + 10 = 4$$

$$3x = -6$$

$$x = -2$$

Thus,  $x = -2$  and  $y = 5$ .

Now, Using Cross Multiplication method:

$$8x + 5y - 9 = 0$$

$$3x + 2y - 4 = 0$$

$$x/(-20 + 18) = y/(-27 + 32) = 1/(16 - 15)$$

$$-x/2 = y/5 = 1/1$$

$$\therefore x = -2 \text{ and } y = 5.$$

**Q.10: Formulate the following problems as a pair of equations, and hence find their solutions:**

**(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.**

**Solution:**

(i) Let us consider,

Speed of boat in still water =  $x$  km/hr

Speed of current =  $y$  km/hr

Now, speed of Ritu, during,

Downstream =  $x + y$  km/hr

Upstream =  $x - y$  km/hr

As per the question given,

$$2(x + y) = 20$$

$$\text{Or } x + y = 10 \dots\dots\dots(1)$$

$$\text{And, } 2(x - y) = 4$$

$$\text{Or } x - y = 2 \dots\dots\dots(2)$$

Adding both the eq.1 and 2, we get,

$$2x = 12$$

$$x = 6$$

Putting the value of  $x$  in eq.1, we get,

$$y = 4$$

Therefore,

Speed of Ritu in still water = 6 km/hr

Speed of current = 4 km/hr

**Q.11: Solve the equations  $x + 2y - 4 = 0$  and  $2x + 4y - 12 = 0$  graphically.**

**Solution:**

Given,

$$x + 2y - 4 = 0 \dots(i)$$

$$2x + 4y - 12 = 0 \dots(ii)$$

From (i),

$$x + 2y = 4$$

$$2y = 4 - x$$

$$y = (4 - x)/2$$

x	0	2	4
y	2	1	0

From (ii),

$$2x + 4y = 12$$

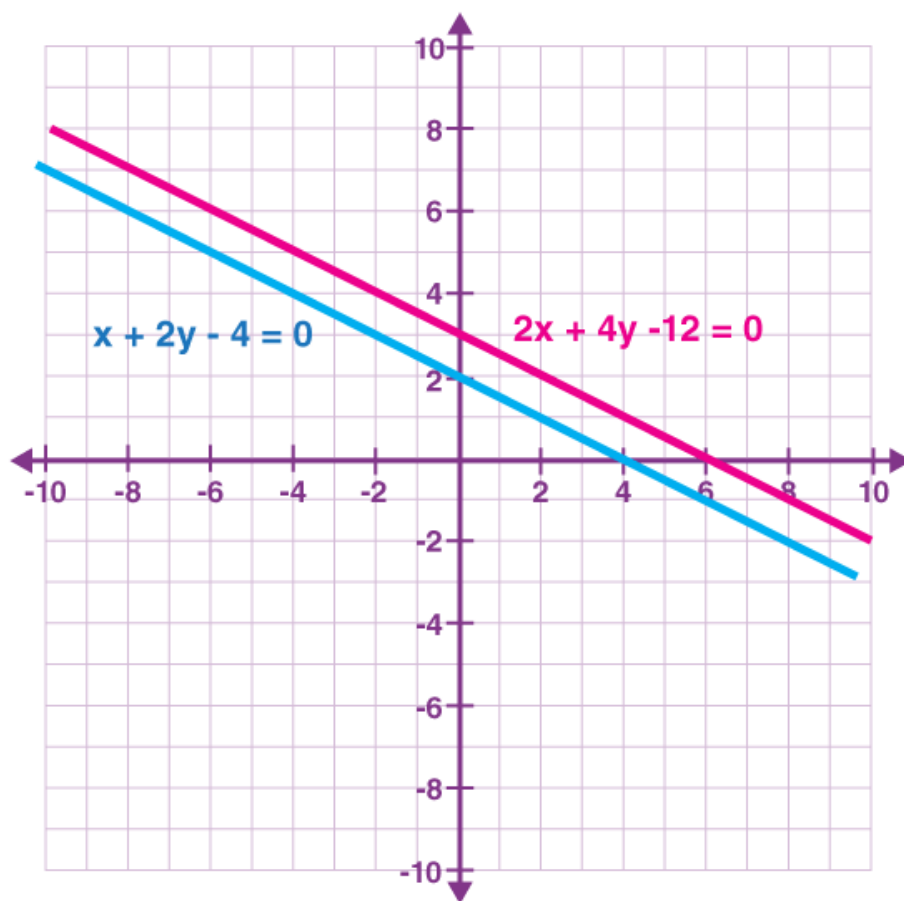
$$x + 2y = 6$$

$$2y = 6 - x$$

$$y = (6 - x)/2$$

x	0	2	4
y	3	2	1

Plotting the points on the graph, we get;



Here, the lines represent the given pair of linear equations are parallel.

Thus, there is no solution to the given pair of linear equations.

**Q.12: Find the value(s) of  $k$  so that the pair of equations  $x + 2y = 5$  and  $3x + ky + 15 = 0$  has a unique solution.**

**Solution:**

Given,

$$x + 2y = 5$$

$$3x + ky + 15 = 0$$

Also, given that the pair of equations has a unique solution.

Comparing the given equations with standard form,

$$a_1 = 1, b_1 = 2, c_1 = -5$$

$$a_2 = 3, b_2 = k, c_2 = 15$$

Condition for unique solution is:

$$a_1/a_2 \neq b_1/b_2$$

$$1/3 \neq 2/k$$

$$k \neq (2)(3)$$

$$k \neq 6$$

Thus, for all real values of  $k$  except 6, the given pair of equations has a unique solution.

**Q.13: Determine graphically the coordinates of vertices of a triangle, the equation of whose sides are given by  $2y - x = 8$ ,  $5y - x = 14$  and  $y - 2x = 1$ .**

**Solution:**

Given,

$$2y - x = 8 \dots (i)$$

$$5y - x = 14 \dots (ii)$$

$$y - 2x = 1 \dots (iii)$$

From (i),

$$2y = x + 8$$

$$y = (x + 8)/2$$

x	-4	0	2
y	2	4	5

From (ii),

$$5y = x + 14$$

$$y = (x + 14)/5$$

x	-4	1	6
y	2	3	4

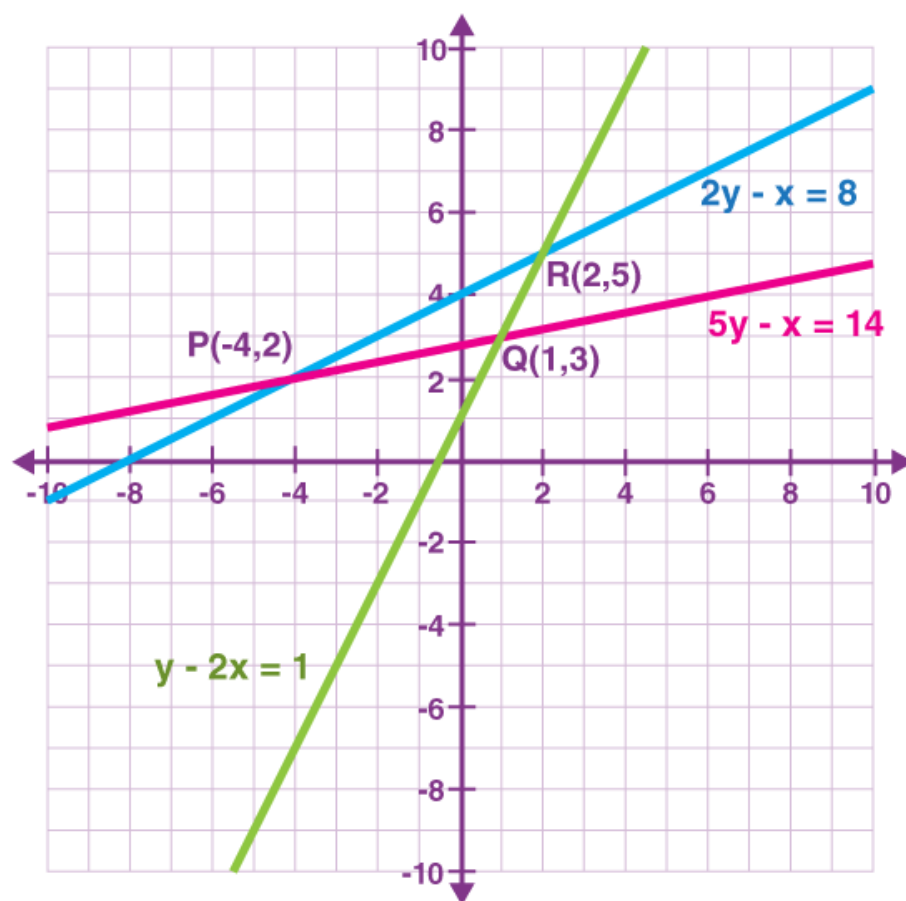
From (iii),

$$y = 2x + 1$$

x	-1	1	2
y	-1	3	5

Let us plot all these points on the graph.





From the graph, we can write the coordinates of vertices of triangle formed are:

$P(-4, 2)$ ,  $Q(1, 3)$ , and  $R(2, 5)$

**Q.14: Use elimination method to find all possible solutions of the following pair of linear equation:**

$$2x + 3y = 8$$

$$4x + 6y = 7$$

**Solution:**

Given,

$$2x + 3y = 8 \dots (i)$$

$$4x + 6y = 7 \dots (ii)$$

Multiply Equation (1) by 2 and Equation (2) by 1 to make the coefficients of x equal.

$$4x + 6y = 16 \dots (iii)$$

$$4x + 6y = 7 \dots (iv)$$

Subtracting (iv) from (iii),

$$4x + 6y - 4x - 6y = 16 - 7$$

$$0 = 9, \text{ it is not possible}$$

Therefore, the pair of equations has no solution.

**Q.15: Solve the following pairs of equations by reducing them to a pair of linear equations:**

$$\frac{1}{2}x + \frac{1}{3}y = 2$$

$$\frac{1}{3}x + \frac{1}{2}y = \frac{13}{6}$$

**Solution:**

Given,

$$\frac{1}{2}x + \frac{1}{3}y = 2$$

$$\frac{1}{3}x + \frac{1}{2}y = \frac{13}{6}$$

Let us assume  $\frac{1}{x} = m$  and  $\frac{1}{y} = n$ , then the equations will change as follows.

$$\frac{m}{2} + \frac{n}{3} = 2$$

$$\Rightarrow 3m + 2n - 12 = 0 \dots (1)$$

$$\frac{m}{3} + \frac{n}{2} = \frac{13}{6}$$

$$\Rightarrow 2m + 3n - 13 = 0 \dots (2)$$

Now, using cross-multiplication method, we get,

$$\frac{m}{(-26 - (-36))} = \frac{n}{(-24 - (-39))} = \frac{1}{(9 - 4)}$$

$$\frac{m}{10} = \frac{n}{15} = \frac{1}{5}$$

$$\frac{m}{10} = \frac{1}{5} \text{ and } \frac{n}{15} = \frac{1}{5}$$

$$\text{So, } m = 2 \text{ and } n = 3$$

$$\frac{1}{x} = 2 \text{ and } \frac{1}{y} = 3$$

$$x = \frac{1}{2} \text{ and } y = \frac{1}{3}$$

**Important 2 Marks Questions for Class 10 Maths Board are as follows-**

**Q.1:** Find the value of  $k$  for which the roots of the quadratic equation

, will have equal value.

**Solution:**

Given,

$$2x^2 + kx + 8 = 0$$

Comparing with the standard form  $ax^2 + bx + c = 0$ ,

$$a = 2, b = k, c = 8$$

Condition for the equal roots is:

$$b^2 - 4ac = 0$$

$$k^2 - 4(2)(8) = 0$$

$$k^2 - 64 = 0$$

$$k^2 = 64$$

$$k = \pm 8$$

**Q.2:** Determine the AP whose third term is 5 and the seventh term is 9.

**Solution:**

Let  $a$  be the first term and  $d$  be the common difference of an AP.

Given,

$$\text{Third term} = 5$$

$$a + 2d = 5 \dots (i)$$

Seventh term = 9

$$a + 6d = 9 \dots (ii)$$

Subtracting (i) from (ii),

$$a + 6d - a - 2d = 9 - 5$$

$$4d = 4$$

$$d = 1$$

Substituting  $d = 1$  in (i),

$$a + 2(1) = 5$$

$$a = 5 - 2 = 3$$

Therefore, the AP is: 3, 4, 5, 6,...

**Q.3:** Find a relation between  $x$  and  $y$  if the points  $A(x, y)$ ,  $B(-4, 6)$  and  $C(-2, 3)$  are collinear.

**Solution:**

Let the given points be:

$$A(x, y) = (x_1, y_1)$$

$$B(-4, 6) = (x_2, y_2)$$

$$C(-2, 3) = (x_3, y_3)$$

If three points are collinear then the area of the triangle formed by these points is 0.

$$\text{i.e. } \left(\frac{1}{2}\right) |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$$

$$\left(\frac{1}{2}\right) |x(6 - 3) + (-4)(3 - y) + (-2)(y - 6)| = 0$$

$$x(3) - 4(3 - y) - 2(y - 6) = 0$$

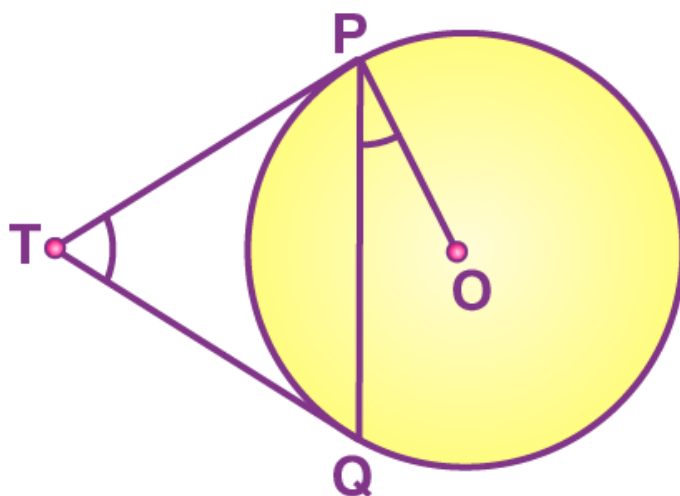
$$3x - 12 + 4y - 2y + 12 = 0$$

$$3x + 2y = 0$$

Or

$$x = -2y/3$$

**Q.4:** In the figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T, prove that  $\angle PTQ = 2\angle OPQ$ .



**Solution:**

Given that two tangents TP and TQ are drawn to a circle with centre O from an external point T

Let  $\angle PTQ = \theta$ .

Now, by using the theorem “the lengths of tangents drawn from an external point to a circle are equal”, we can say  $TP = TQ$ . So, TPQ is an isosceles triangle.

Thus,

$$\angle TPQ = \angle TQP = \frac{1}{2} (180^\circ - \theta) = 90^\circ - \left(\frac{1}{2}\right) \theta$$

By using the theorem, “the tangent at any point of a circle is perpendicular to the radius through the point of contact”, we can say  $\angle OPT = 90^\circ$

Therefore,

$$\angle OPQ = \angle OPT - \angle TPQ = 90^\circ - [90^\circ - (\frac{1}{2}) \theta]$$

$$\angle OPQ = (\frac{1}{2})\theta$$

$$\angle OPQ = (\frac{1}{2}) \angle PTQ$$

$$\Rightarrow \angle PTQ = 2 \angle OPQ.$$

Hence proved.

**Q.5:** A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of  $60^\circ$  at its centre. Find the radius of the circle. [Use  $\pi = 22/7$ ]

**Solution:**

Length of the arc = Length of wire

$$(\theta/360^\circ) 2\pi r = 22$$

$$(60^\circ/360^\circ) \times 2 \times (22/7) \times r = 22$$

$$(\frac{1}{6}) \times (2/7) \times r = 1$$

$$r = (7/2) \times 6$$

$$r = 21$$

Therefore, the radius of the circle is 21 cm.

**Q.6:** If a number  $x$  is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3, what is the probability that  $x^2 \leq 4$ ?



**Solution:**

Sample space =  $S = \{-3, -2, -1, 0, 1, 2, 3\}$

$$n(S) = 7$$

Let E be the event of choosing a number x such that  $x^2 \leq 4$ .

i.e.  $x \leq \pm 2$

$$E = \{-2, -1, 0, 1, 2\}$$

$$n(E) = 5$$

$$P(E) = n(E)/n(S) = 5/7$$

Hence, the required probability is  $5/7$ .

**Q.7:** The larger of two supplementary angles exceeds the smaller by  $18^\circ$ . Find the angles.

**Solution:**

Let x and  $(x + 18^\circ)$  be the supplementary angles.

That means,

$$x + (x + 18^\circ) = 180^\circ$$

$$2x = 180^\circ - 18^\circ$$

$$2x = 162^\circ$$

$$x = 162^\circ/2$$

$$x = 81^\circ$$

$$\text{Now, } x + 18^\circ = 81^\circ + 18^\circ = 99^\circ$$

Therefore, the supplementary angles are  $81^\circ$  and  $99^\circ$ .

**Q.8:** Find the mean of the following distribution:

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	5	10	10	7	8

**Solution:**

Class	Frequency ( $f_i$ )	Midpoint ( $x_i$ )	$f_i x_i$
3 – 5	5	4	20
5 – 7	10	6	60
7 – 9	10	8	80
9 – 11	7	10	70
11 – 13	8	12	96
Total	$\Sigma f_i = 40$	$\Sigma x_i = 40$	$\Sigma f_i x_i = 326$

$$\text{Mean} = \Sigma f_i x_i / \Sigma f_i$$

$$= 326/40$$

$$= 8.15$$

Therefore, the mean of the given distribution is 8.15.

**Q.9:** Given that  $\sqrt{2}$  is irrational, prove that  $(5 + 3\sqrt{2})$  is an irrational number.

**Solution:**

Let  $(5 + 3\sqrt{2})$  be a rational number.

$$5 + 3\sqrt{2} = p/q \text{ (Where } q \neq 0 \text{ and } p \text{ and } q \text{ are co-prime numbers)}$$

$$3\sqrt{2} = (p/q) - 5$$

$$3\sqrt{2} = (p - 5q)/q$$

$$\sqrt{2} = (p - 5q)/3q$$

$p$  and  $q$  are integers and  $q \neq 0$

Thus,  $(p - 5q)/3q$  is rational number.

Also,  $\sqrt{2}$  is a rational number.

However, it is given that  $\sqrt{2}$  is an irrational number.

This is a contradiction, and thus, our assumption that  $(5 + 3\sqrt{2})$  be a rational number is wrong.

That means  $(5 + 3\sqrt{2})$  is an irrational number.

Hence proved.

**Q.10:** Find the value of  $p$ , for which one root of the quadratic equation  $px^2 - 14x + 8 = 0$  is 6 times the other.

**Solution:**

Given quadratic equation is:

$$px^2 - 14x + 8 = 0$$

Let  $\alpha$  and  $6\alpha$  be the roots of the given quadratic equation.

Sum of the roots =  $-\text{coefficient of } x / \text{coefficient of } x^2$

$$\alpha + 6\alpha = -(-14)/p$$

$$7\alpha = 14/p$$

$$\alpha = 2/p \dots (i)$$

Product of roots =  $\text{constant term} / \text{coefficient of } x^2$

$$(\alpha)(6\alpha) = 8/p$$

$$6\alpha^2 = 8/p$$

Substituting  $\alpha = 2/p$  from (i),

$$6 \times (2/p)^2 = 8/p$$

$$24/p^2 = 8/p$$

$$3/p = 1$$

$$p = 3$$

Therefore, the value of  $p$  is 3.

**Important 3 Marks Questions and Solutions for Class 12 Maths Board are as follows-**

**Question 1- If  $m$ th term of an A.P. is**

**and  $n$ th term is**

**, then find the sum of its first  $mn$  terms.**

**Solution:**

Let  $a$  be the first term and  $d$  be the common difference of an AP.

Given,

$$a_m = 1/n$$

$$\text{i.e. } a + (m - 1)d = 1/n$$

$$n[a + (m - 1)d] = 1$$

$$an + mnd - nd = 1 \dots (i)$$

And

$$a_n = 1/m$$

$$a + (n - 1)d = 1/m$$

$$m[a + (n - 1)d] = 1$$

$$am + mnd - md = 1 \dots (ii)$$

From (i) and (ii),

$$an + mnd - nd = am + mnd - md$$

$$an - am = nd - md$$

$$a(n - m) = (n - m)d$$

$$a = d$$

Substituting  $a = d$  in equation (i),

$$dn + mnd - nd = 1$$

$$mnd = 1$$

$$d = 1/mn$$

Therefore,  $a = 1/mn$

Sum of first  $mn$  terms is:

$$S_{mn} = (mn/2)[2(1/mn) + (mn - 1)(1/mn)]$$

$$= (mn/2)[(2/mn) + (mn - 1)/mn]$$

$$= (mn/2mn)[2 + mn - 1]$$

$$= (1/2)(mn + 1)$$

Hence, the sum of first  $mn$  terms  $= (mn + 1)/2$ .

**Question 2- A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from**

**to**

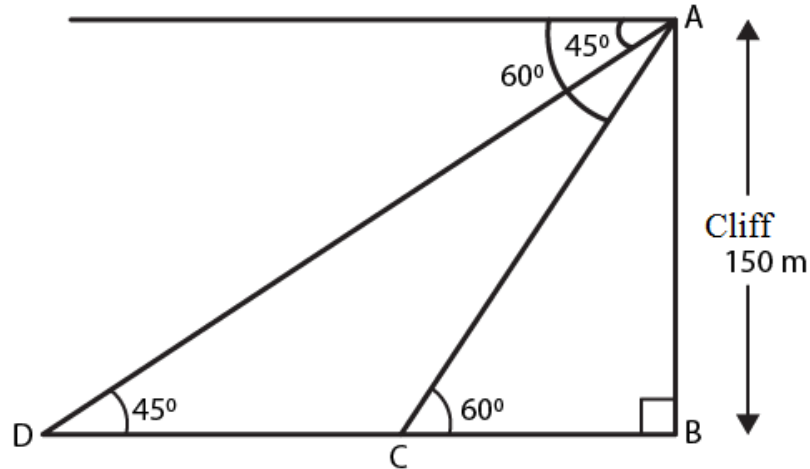
**in 2 minutes. Find the speed of the boat in m/h.**

**Solution:**

Let AB be the height of the cliff.

AB = 150 m

C and D be the position of a boat.



Boat is moved from C to D in 2 minutes =  $\frac{2}{60}$  hr =  $\frac{1}{30}$  hr

In right triangle ABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = 150/BC$$

$$BC = 150/\sqrt{3} \text{ m}$$

In right triangle ABD,

$$\tan 45^\circ = AB/AD$$

$$1 = 150/(BC + CD)$$

$$(150/\sqrt{3}) + CD = 150$$

$$CD = 150 - (150/\sqrt{3})$$

$$CD = (150\sqrt{3} - 150)/\sqrt{3}$$

$$= 150(\sqrt{3} - 1)/\sqrt{3} \text{ m}$$

Distance CD = Speed  $\times$  time

$$150(\sqrt{3} - 1)/\sqrt{3} = \text{Speed} \times (1/30)$$

$$\text{Speed} = 30 \times [150(\sqrt{3} - 1)/\sqrt{3}]$$

$$= 1500\sqrt{3}(\sqrt{3} - 1) \text{ m/h}$$

**Question 3- Two different dice are thrown together. Find the probability that the numbers obtained**

**(i) have a sum less than 7**

**(ii) have a product less than 16**



**(iii) is a doublet of odd numbers.**

**Solution:**

Given,

Two different dice are thrown together.

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$$

$$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$$

$$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$$

$$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$$

$$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$$

$$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$$\text{Total number of outcomes} = n(S) = 36$$

(i) have a sum less than 7

Let  $E_1$  be the event of getting the numbers on dice whose sum is less than 7.

$$E_1 = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)\}$$

$$n(E_1) = 15$$

$$P(E_1) = n(E_1)/n(S) = 15/36 = 5/12$$

(ii) have a product less than 16

Let  $E_2$  be the event of getting the numbers on dice whose product is less than 16.

$$E_2 = \{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2) \}$$

$$n(E_2) = 25$$

$$P(E_2) = n(E_2)/n(S) = 25/36$$

(iii) is a doublet of odd numbers.

Let  $E_3$  be the event of getting a doublet of odd numbers.

$$E_3 = \{ (1, 1), (3, 3), (5, 5) \}$$

$$P(E_3) = n(E_3)/n(S) = 3/36 = 1/12$$

**Question 4- The area of a triangle is 5 sq units. Two of its vertices are (2,1) and (3,-2). If the third vertex is**

**, then find the value of y.**

**Solution:**

Let A, B, and C be the vertices of a triangle.

$$A(2, 1) = (x_1, y_1)$$

$$B(3, -2) = (x_2, y_2)$$

$$C(7/2, y) = (x_3, y_3)$$

Given,

Area of the triangle is 5 sq.units.

$$(1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 5$$

$$(1/2)|2(-2 - y) + 3(y - 1) + (7/2)(1 + 2)| = 5$$

$$|-4 - 2y + 3y - 3 + (7/2)(3)| = 10$$

$$y - 7 + (21/2) = 10$$

$$y = 10 + 7 - (21/2)$$

$$y = 17 - (21/2)$$

$$y = (34 - 21)/2$$

$$y = 13/2$$

Therefore, the value of y is 13/2.

### Question 5- The

**th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with an internal radius of 10 cm. Find the height of water in the cylindrical vessel.**

**Solution:**

Given,

Height of conical vessel =  $h = 24$  cm

Radius of conical vessel =  $r = 5$  cm

Volume of water =  $(3/4) \times$  Volume of conical vessel

$$= (3/4) \times (1/3)\pi r^2 h$$

$$= (3/4) \times (1/3) \times (\pi \times 5 \times 5 \times 24)$$

$$= 150\pi \text{ cm}^3$$

Let  $H$  be the height of a cylindrical vessel.

Radius of cylindrical vessel =  $R = 10$  cm {given}

From the given,

Volume of cylindrical vessel = Volume of water

$$\pi R^2 H = 150\pi$$

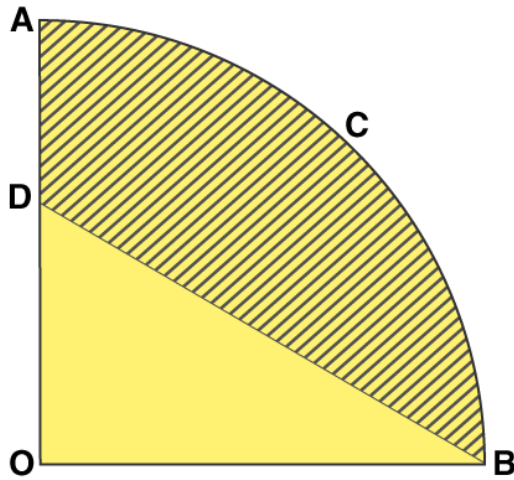
$$R^2 H = 150$$

$$10 \times 10 \times H = 150$$

$$H = 1.5 \text{ cm}$$

Therefore, the height of a cylindrical vessel.

**Question 6- In the given figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the shaded region.**



**Solution:**

Given,

Radius of the quadrant ( $r$ ) = 3.5 cm

$$\text{Area of quadrant} = \left(\frac{1}{4}\right)\pi r^2$$

$$= \left(\frac{1}{4}\right) \times \left(\frac{22}{7}\right) \times 3.5 \times 3.5$$

$$= 9.625 \text{ cm}^2$$

$$\text{Area of triangle BOD} = \left(\frac{1}{2}\right) \times OB \times OD$$

$$= \left(\frac{1}{2}\right) \times 3.5 \times 2$$

$$= 3.5 \text{ cm}^2$$

$$\text{Area of the shaded region} = \text{Area of quadrant} - \text{Area of triangle BOD}$$

$$= 9.625 - 3.5$$

$$= 6.125 \text{ cm}^2$$

**Question 7- If the equation**

**has equal roots then show that**

.

**Solution:**

Given,

$$(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$$

Comparing with the standard form of a quadratic equation  $ax^2 + bx + c = 0$ ,

$$a = (1 + m^2)$$

$$b = 2mc$$

$$c = (c^2 - a^2)$$

Given that the quadratic equation has equal roots.

So,  $b^2 - 4ac = 0$  {determinant is equal to 0}

$$(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$

$$4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$

$$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$$

$$4c^2 = 4a^2m^2 + 4a^2$$

$$4c^2 = 4a^2(m^2 + 1)$$

$$c^2 = a^2(1 + m^2)$$

**Question 8- Find the sum of n terms of the series**

**Solution:**

Given series:

$$[4 - (1/n)] + [4 - (2/n)] + [4 - (3/n)] + \dots \text{ up to } n \text{ terms}$$

$$= (4 + 4 + 4 + \dots \text{ up to } n \text{ terms}) - [(1/n) + (2/n) + (3/n) + \dots \text{ up to } n \text{ terms}]$$

$$= 4n - (1/n)[1 + 2 + 3 + \dots \text{ up to } n \text{ terms}]$$

$$= 4n - (1/n)[n(n + 1)/2] \quad \{\text{sum of first } n \text{ natural numbers} = n(n + 1)/2\}$$

$$= 4n - (n + 1)/2$$

$$= (8n - n - 1)/2$$

$$= (7n - 1)/2$$

Therefore, the sum of the given series is  $(7n - 1)/2$ .

**Question 9- Prove that:  $(\sec A - \cos A) \cdot (\cot A + \tan A) = \tan A \cdot \sec A$**

**Solution:**

$$\text{LHS} = (\sec A - \cos A)(\cot A + \tan A)$$

$$= [(1/\cos A) - \cos A] [(\cos A/\sin A) + (\sin A/\cos A)]$$

$$= [(1 - \cos^2 A)/\cos A] [(\cos^2 A + \sin^2 A)/(\sin A \cos A)]$$

$$= (\sin^2 A / \cos A) [1/(\sin A \cos A)]$$

$$= \sin A / (\cos A \cos A)$$

$$= (\sin A / \cos A) (1/\cos A)$$

$$= \tan A \sec A$$

$$= \text{RHS}$$

$$\text{Thus, } (\sec A - \cos A) \cdot (\cot A + \tan A) = \tan A \cdot \sec A$$

Hence proved.

**Question 10-** The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f.

Daily pocket allowance (in Rs.)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of children	7	6	9	13	f	5	4

**Solution:**

Class interval	Number of children ( $f_i$ )	Mid-point ( $x_i$ )	$f_i x_i$
11 – 13	7	12	84



13 – 15	6	14	84
15 – 17	9	16	144
17 – 19	13	18	234
19 – 21	f	20	20f
21 – 23	5	22	110
23 – 25	4	24	96
Total	$\sum f_i = 44 + f$		$\sum f_i x_i = 752 + 20f$

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{(752 + 20f)}{(44 + f)}$$

$$\text{Given that, } \frac{(752 + 20f)}{(44 + f)} = 18$$

$$752 + 20f = 18(44 + f)$$

$$752 + 20f = 792 + 18f$$

$$20f - 18f = 792 - 752$$

$$2f = 40$$

$$f = 20$$

Therefore, the missing frequency =  $f = 20$

**Important 4 Marks Questions and Solutions for Class 10 Maths Board are as follows-**

**Question 1-** Solve for x:

**Solution:**

Taking LCM and cross multiplying the terms,

$$x[x - 1 + 2x - 4] = 6[x^2 - 2x - x + 2]$$

$$x(3x - 5) = 6(x^2 - 3x + 2)$$

$$6x^2 - 18x + 12 - 3x^2 + 5x = 0$$

$$3x^2 - 13x + 12 = 0$$

$$3x^2 - 9x - 4x + 12 = 0$$

$$3x(x - 3) - 4(x - 3) = 0$$

$$(3x - 4)(x - 3) = 0$$

$$3x - 4 = 0, x - 3 = 0$$

$$x = 4/3, 3$$

**Question 2-**

Find the sum of first 24 terms of an A.P. whose nth term is given by

.

**Solution:**

Given,

$$a_n = 3 + 2n$$

Substituting  $n = 1, 2, 3, \dots$

When  $n = 1$ ,

$$a_1 = 3 + 2(1) = 3 + 2 = 5$$

When  $n = 2$ ,

$$a_2 = 3 + 2(2) = 3 + 4 = 7$$

When  $n = 3$ ,

$$a_3 = 3 + 2(3) = 3 + 6 = 9$$

Thus, the AP is: 5, 7, 9,...

Here,

First term =  $a = 5$

Common difference =  $d = 2$

We know that,

Sum of first  $n$  terms of an AP =  $S_n = \frac{n}{2} [2a + (n - 1)d]$

Now,

$$S_{24} = (24/2) [2(5) + (24 - 1)(2)]$$

$$= 12[10 + 23(2)]$$

$$= 12[10 + 46]$$

$$= 12 \times 56$$

$$= 672$$

Therefore, the sum of the first 24 terms of the given AP is 672.

### **Question 3-**

A bucket, is in the form of a frustum of a cone whose height is 42 cm and the radii of its circular ends are 30 cm and 10 cm. Find the amount of milk (in litres) which this bucket can hold. If the milkman sells the milk at the rate of Rs. 40 per litre, what amount he will get from the sale?

If the milkman sells half the milk at less rate to the economically weaker section of society, what value he exhibits by doing this?

### **Solution:**

Given that a bucket is in the form of a frustum of a cone with the following dimensions.

Height =  $h$  = 42 cm

$R$  = 30 cm

$$r = 10 \text{ cm}$$

$$\text{Volume of frustum of a cone} = \left(\frac{1}{3}\right)\pi h[R^2 + r^2 + Rr]$$

Substituting the values,

$$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 42 \times [(30)^2 + (10)^2 + (30)(10)]$$

$$= 44 \times [900 + 100 + 300]$$

$$= 44 \times 1300$$

$$= 572000 \text{ cm}^3$$

$$= (572000/1000) \text{ litres} \{\text{since } 1 \text{ cm}^3 = 1/1000 \text{ litres}\}$$

$$= 57.2 \text{ litres}$$

Therefore, the bucket can hold 57.2 litres of milk.

Also,

$$\text{Cost of 1 litre milk} = \text{Rs. } 40 \text{ (given)}$$

$$\text{So, the cost of 57.2 litres of milk} = 57.2 \times \text{Rs. } 40 = \text{Rs. } 2288$$

Hence, the milkman can get Rs. 2288 from selling the milk in the bucket.

If the milkman sells half the milk at less rate to the economically weaker section of society, he shows the values of help, kindness, and empathy.

**Question 4-**

If the coordinates of two points are A(3,4), B (5,-2) and a point P (x,5) is such that PA = PB, then find the area of

.

**Solution:**

Given points:

A(3, 4), B(5, -2) and P(x, 5)

Also,

$$PA = PB$$

Squaring on both sides,

$$PA^2 = PB^2$$

Using distance formula,  $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ ;

$$(x - 3)^2 + (5 - 4)^2 = (x - 5)^2 + (5 + 2)^2$$

$$x^2 + 9 - 6x + 1 = x^2 + 25 - 10x + 49$$

$$10 - 6x = 74 - 10x$$

$$\Rightarrow 10x - 6x = 74 - 10$$

$$\Rightarrow 4x = 64$$

$$\Rightarrow x = 16$$

$$\Rightarrow P = (x, 5) = (16, 5)$$

Let,

$$P(16, 5) = (x_1, y_1), A(3, 4) = (x_2, y_2), B(5, -2) = (x_3, y_3)$$

$$\text{Area of triangle PAB} = (1/2)|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= (1/2)|16(4 + 2) + 3(-2 - 5) + 5(5 - 4)|$$

$$= (1/2)|16(6) + 3(-7) + 5(1)|$$

$$= (1/2)|96 - 21 + 5|$$

$$= (1/2) \times 80$$

$$= 40 \text{ square units}$$

### **Question 5-**

A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys so formed.

### **Solution:**

Given,

Height of cylinder = H = 15 cm

Diameter = 12 cm

Radius of cylinder =  $R = 12/2 = 6$  cm

Radius of cone = 3 cm

Height of cone =  $h = 9$  cm

Let  $n$  be the number of toys in the shape of a cone made.

So,

Volume of cylinder =  $n \times$  volume of a cone

$$\pi R^2 H = (1/3) \pi r^2 h$$

$$6 \times 6 \times 15 = n \times (1/3) \times 3 \times 3 \times 9$$

$$n = (6 \times 6 \times 15 \times 3) / (3 \times 3 \times 9)$$

$$n = 20$$

### **Question 6-**

From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed. The remaining cards are mixed and a card is drawn at random. Find the probability that the drawn card is

(i) a black Queen

(ii) a card of red colour

(iii) a Jack of black colour



(iv) a face card

**Solution:**

We know that,  $P(\text{an event}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

Given,

From a pack of 52 playing cards, Jacks and Kings of red colour and Queens and Aces of black colour are removed.

So, the total number of outcomes =  $52 - (2 + 2 + 2 + 2) = 52 - 8 = 44$

(i) a black Queen

Number of black queen cards in the available cards = 0

$P(\text{getting a black queen}) = \frac{0}{44} = 0$

(ii) a card of red colour

Number of red colour cards =  $26 - 4 = 22$

$P(\text{getting a card of red colour}) = \frac{22}{44} = \frac{1}{2}$

(iii) a Jack of black colour

Number of black Jack cards = 2

$P(\text{getting a Jack of black colour}) = \frac{2}{44} = \frac{1}{22}$

(iv) a face card

$$\text{Number of face cards} = 12 - 6 = 6$$

$$P(\text{getting a face card}) = 6/44 = 3/22$$

### **Question 7-**

From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some vertically over the top the building is

and

respectively. Find the height of the helicopter above the ground.

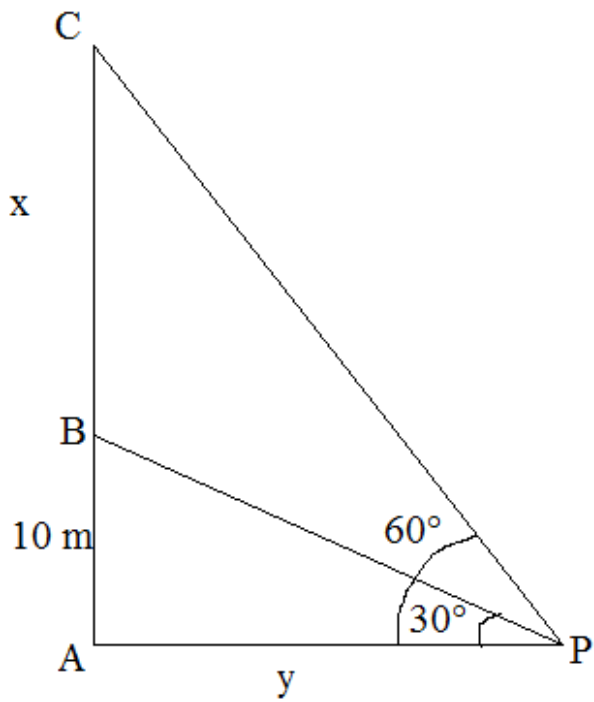
### **Solution:**

Let P be the position of a point and C be the position of a helicopter.

Height of the building AB = 10 m

$$AC = x \text{ m}$$

$$PA = y \text{ m}$$



In right triangle PAB,

$$\tan 30^\circ = AB/PA$$

$$1/\sqrt{3} = 10/y$$

$$y = 10\sqrt{3} \text{ m}$$

In right triangle PAC,

$$\tan 60^\circ = AC/PA$$

$$\sqrt{3} = (x + 10)/10\sqrt{3}$$

$$10\sqrt{3} \sqrt{3} = x + 10$$

$$x = 30 - 10 = 20$$

Therefore, the height of helicopter from the ground =  $x + 10 = 20 + 10 = 30$  m

### **Question 8-**

Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm.

Construct tangents to each circle from the centre of the other circle.

### **Solution:**

#### **Steps of Construction:**

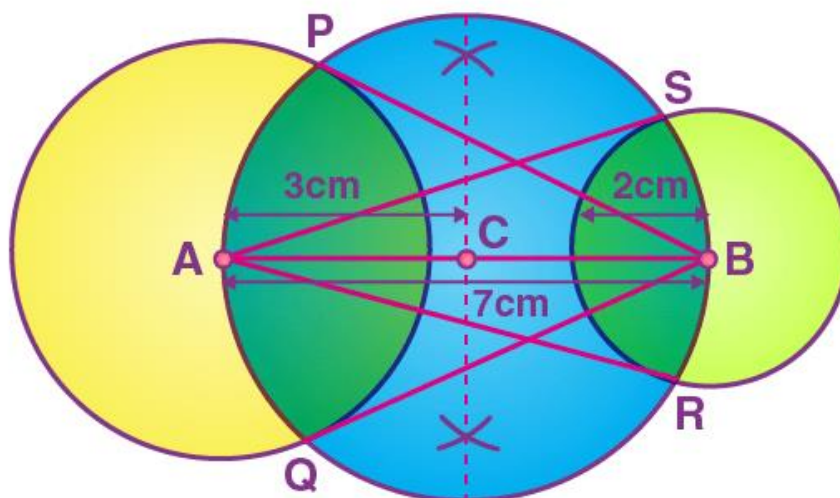
**Step 1:** Draw a line segment AB of 7 cm.

**Step 2:** Taking A and B as centres, draw two circles of 3 cm and 2 cm radius respectively.

**Step 3:** Bisect the line AB. Let the midpoint of AB be C.

**Step 4:** Taking C as centre, draw a circle of radius AC which intersects the two circles at point P, Q, R and S.

**Step 5:** Join BP, BQ, AS and AR.



Here, BP, BQ, AS and AR are the tangents to the circles.

### Question 9-

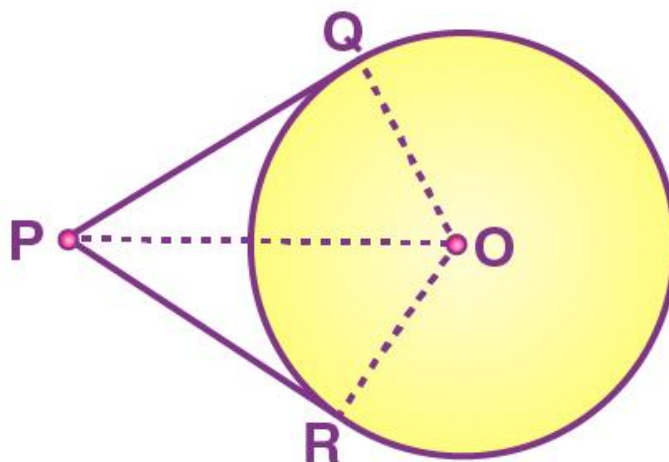
Prove that the lengths of tangents drawn from an external point to a circle are equal.

### Solution:

Consider a circle with the centre "O" and P is the point that lies outside the circle. Hence, the two tangents formed are PQ and PR.

We need to prove:  $PQ = PR$ .

To prove the tangent PQ is equal to PR, join OP, OQ and OR. Hence,  $\angle OQP$  and  $\angle ORP$  are the right angles.



Therefore,  $OQ = OR$  (Radii)

$OP = OP$  (Common side)

By using the RHS rule, we can say,  $\triangle OQP \cong \triangle ORP$ .

Thus, by using the CPCT rule, the tangent  $PQ = PR$ .

Hence proved.

### Question 10-

The Life insurance agent found the following data for the distribution of ages of 100 policyholders. Calculate the median age, if policies are given only to the persons whose age is 18 years onwards but less than the 60 years.

Age (in years)	Number of policyholders
----------------	-------------------------

Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

**Solution:**

Let us calculate the class intervals and the corresponding frequencies for the given data.

Class interval	Frequency	Cumulative frequency
15 – 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45

35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100

$$N = 100$$

$$N/2 = 100/2 = 50$$

The cumulative frequency greater than and nearer to 50 is 78 which belongs to the class interval 35 – 40.

So, the median class is 35 – 40.

Frequency of the median class =  $f = 33$

Lower limit of median class =  $l = 35$

Cumulative frequency of the class preceding the median class =  $cf = 45$

Class size =  $h = 5$

$$\text{Median} = l + [(N/2 - cf)/f] \times h$$

$$= 35 + [(50 - 45)/33] \times 5$$

$$= 35 + (25/33)$$



$$= 35 + 0.75$$

$$= 35.75 \text{ (approx.)}$$

Therefore, the median age = 35.75 years.